PERFORMANCE PARAMETERS FOR GAS GENERATOR COMPARISON

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THESIS

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by

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ABSTRACT

Methods for comparing gas turbine engines where the thermodynamic cycle begins and ends in the atmosphere are well defined and documented. No such comparison technique(s) exists for the gas generator or core portion of the engine. The term gas generator or core refers to the high pressure compressor and turbine, and the combustor.

This thesis formulates gas generator performance parameters, develops methods of testing and data reduction necessary to obtain these parameters, establishes criteria for comparing two gas generators, and develops an analytical model to test the validity of the comparison method.

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Symbol	
a	speed of sound (ft/sec)
c_i	constant used in determination of engine weight (lb/lbm/sec)
C_p	specific heat at constant pressure (BTU/lb°R)
F/A	thrust per unit area of engine inlet (lb/ft^2)
F/m	specific thrust (lb/lbm/sec)
F/W_e	thrust per lb of engine weight (dimensionless)
f	fuel air ratio (lb fuel/lb air)
g	acceleration due to gravity (32.17 ft/sec^2)
HP/m	specific gas horsepower (ft-lb)/lbm
N	Revolutions per minute (1/min)
L	characteristic length (ft)
m_a	air mass flow rate (lbm/sec)
m_c	air mass flow rate through the core (lbm/sec)
m_f	fuel mass flow rate (lbm/sec)
m_{fan}	air mass flow rate through the fan, by-pass air (lbm/sec)
m_t	total air mass flow rate, $m_c + m_{fan}$ (lbm/sec)
P	Pressure (lb/in^2 or ATMS)
Q	heating value (BTU/lb)
R	ratio of gas generator weight to total engine weight (dimensionless)
ΔS	change in entropy (BTU/lb°R)
T	temperature (°R)
U_{80}	Ratio of Velocity$_8$/Velocity$_0$ (dimensionless)
U_{90}	Ratio of Velocity$_9$/Velocity$_0$ (dimensionless)
β	by-pass ratio; air by-passed/core air (dimensionless)

δ Corrected pressure, P/P_{STD} (dimensionless)

γ specific heat ratio; C_p/C_v (dimensionless)

η_c compressor efficiency

η_b combustor efficiency

η_t turbine efficiency

θ corrected temperature, T/T_{STD} (dimensionless)

τ_b combustor temperature ratio, T_{t_4}/T_{t_3} (dimensionless)

τ_c compressor temperature ratio T_{t_3}/T_{t_2} (dimensionless

τ_c' fan temperature ratio, T_{t_8}/T_{t_0} (dimensionless)

τ_r ram temperature ratio, T_{t_2}/T_{t_0} (dimensionless)

τ^* ratio of TIT/inlet temperature, T_{t_4}/T_0 (dimensionless)

π_b combustor pressure ratio, P_{t_4}/P_{t_3} (dimensionless)

π_c compressor pressure ratio, P_{t_3}/P_{t_2} (dimensionless)

π_t turbine pressure ratio, $P_{t_{4.5,5}}/P_{t_4}$ (dimensionless)

Subscripts

t total conditions

0-9 various sections of a gas turbine engine; see Figures 1 and 2

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I. INTRODUCTION

Recent achievements recorded by the gas generator concept and its predictable prominent role in future propulsion systems dictated the need to develop a method to compare the relative merits of various gas generators. Methods for comparing gas turbine engines where the thermodynamic cycle begins and ends in the atmosphere are well defined and documented. See for example References 1 and 2. No such method(s) has been devised for the gas generator, per se.

This paper attempts to develop a logical and consistent, yet simple technique that will permit meaningful comparison of different cores. Specifically the following five areas were addressed: (1) identification of the problems inherent to gas generator comparison, (2) formulation of meaningful core performance parameters, (3) development of a valid, yet simple, method of testing and data reduction to obtain these parameters, (4) establishment of criteria for comparison of cores, (5) development of an analytical model to test the suitability of the parameters, testing procedure and comparison criteria.

II. PHILOSOPHY OF THE GAS GENERATOR

A. DEFINITION AND DESCRIPTION

Before any of the desired analysis could be conducted, it was necessary to clearly define the term gas generator or core. Throughout this paper the term core is used synonomously with gas generator.

The core of an aircraft gas turbine consists of the high pressure compressor, combustor, and the high pressure turbine. Thus essentially the core is the basic building block of a gas turbine. For example, the addition of a fan produces a low SFC turbofan; one such engine is the TF-34 for the S-3. Addition of a low pressure spool and a low pressure turbine results in a high thrust-to-weight turbojet. Figures 1 and 2 depict the above examples. The core in these diagrams is clearly indicated by the dashed lines. Temperature and pressure subscripts used throughout this paper refer to the stations as shown in Figures 1 and 2. Additional subscripts when utilized are referred to a specific figure.

B. OVERVIEW

At the start of the investigation the following question was posed, "Why is the core approach used?" The following brief synopsis explains the philosophy behind the gas generator concept and in so doing aids in the selection of possible comparative techniques.

The initial objective of the gas generator program was to obtain technological advance at lower cost. It was realized at an early date that the most challenging technological problems were focused in the

gas generator portion of the engine. The gas generator is subjected to not only the highest temperature and pressures, but it experiences the highest aerodynamic and thermal stress. It was reasoned that significant savings in development time and money could be saved via an approach that concentrated on the technology associated with the gas generator. Errors could easily be corrected and bad ideas discarded while investment time and dollars were at a lower level.

The gas generator development concept has progressed through several general and specific objectives to its present-day form as the "Advanced Turbine Engine Gas Generator" (ATEGG) program. The objective of ATEGG is to provide a state-of-the-art gas generator applicable to diverse military applications.

Since its conception in 1965 the ATEGG program has proven the wisdom of this design philosophy. Current-day gas generators far surpass their predecessors in virtually all aspects of performance. For example current ATEGG cores require significantly less compressor stages to produce a pressure ratio comparable to the original ATEGG generator. Similarly the number of turbine stages required has seen a reduction. Turbine inlet temperatures (TIT) have increased significantly. For additional information concerning current developments in the ATEGG program see References 3 and 4.

C. DESIGN PHILOSOPHY

In order to gain an insight into the formulation of a core comparison technique, a brief review of the core design philosophy was conducted.

Current gas generators are developed under the philosophy that the core with suitable scaling and the addition of such appendages as fans, additional compressor stages, afterburners etc. would provide suitable powerplants for a variety of missions. Figure 3 illustrates some of these missions via a sketch of the associated power plant.

Each mission is defined in terms of measurable or computed performance parameters i.e.; F/m, F/A, SFC etc. or a combination thereof. A thermodynamic cycle is then generated to optimize each mission. Tradeoffs between the relative value of performance parameters finally yield a core that represents a common denominator for all of the missions considered. From the preceding analysis, core cycle parameters e.g. maximum combustor temperature rise, and maximum TIT can be chosen that will fit a variety of missions. For example, Figure 4 illustrates typical thermodynamic cycles traced by engines used in three diverse missions. Partial data used in the construction of this figure were obtained from Reference 5. The cycles characterize a low by-pass turbofan (TF-30), high by-pass turbofan (TF-39) and a supersonic turbojet (J79), run at 100% of rated revolutions per minute (RPM) and sea level static (SLS) conditions. It should be noted that due to lack of data, engines were assumed to be 100% efficient. The similarity in the cycles indicated that a common core, with modifications to account for current technology, might prove suitable to power all three engines.

III. GAS GENERATOR COMPARISON PROBLEMS

In the development of a gas generator comparison technique, certain problems inherent to the core's position in the engine became apparent. Three of the most significant problem areas are analyzed via an investigation of the associated thermodynamic cycle. Early recognition of the exact nature of these difficulties would have simplified the process of selecting meaningful core performance parameters, a method of testing and comparison criteria. Unfortunately these problems were, in many cases, of such a subtle nature that early identification of their exact ramifications was difficult.

A. ENGINE APPLICATION VS. TEST STAND CYCLE

To illustrate the differences that exist between the thermodynamic cycle traced by a core running at SLS conditions and the cycle traced by the same core integrated into an engine, the following investigation was conducted.

Three diverse applications were simulated by specifying a different total compression ratio for each application. The compression ratios selected were arbitrary in that they did not represent any specific application. The core itself was characterized by a maximum turbine inlet temperature of 3000°R and a compression ratio of ten. These values were felt to be representative of the current state of the art, for a gas generator. All three engines were rated at the same TIT (that of the core) and all engine components were assumed to be 100% efficient, so as to facilitate comparison. The cycle traced by the engines running at SLS and 100% RPM are shown in Figure 5. Figure 6

shows the core "broken out of" each cycle (e.g. DHIM, CGIN, BFKO) and compared with the core SLS cycle (AELP). For each application different values of inlet temperature and pressures (points B, C, and D in Figure 6) occurred at core inlet. Additionally there was found to be a wide variance in the values of turbine exhaust temperature and pressures (M, N and O).

Thus unlike a gas turbine engine where input and output conditions are well defined (e.g. atmosphere to atmosphere; thereby simplifying comparison) the same core used in different applications requires that a new and unique input condition be defined for each application. This lack of commonality of input conditions serves to make comparison difficult.

When two different cores are considered, the dissimilarity becomes even more pronounced. Take the case where the mission to be optimized requires a maximumization of F/m in a turbojet. Two cores with the same compression ratio (10.0) and same efficiency (100%), but with different TIT (3000°R vs 2500°R) will be employed as the building block to optimize the engine. According to Reference 6, F/m is a maximum when the split between the compressor and the combustor is such that,

$$\tau_c = \sqrt{\tau^*}/\tau_r \qquad (1)$$

Figure 7 shows the cycle traced by the core portion of these engines (BDAI, CEFG). Not only do the cycles differ in turbine exhaust conditions (I,G) as expected, but inlet points are seen to vary (B,C). As cited in the previous example a common input condition does not exist for these two cores.

B. CORE COMPRESSION RATIO

Two cores with the same TIT but different compression ratios (10.0 and 6.0) are seen in Figure 8 to have produced unique compressor inlet temperatures (B,C) and turbine outlet temperatures (I,H) in the process of optimizing a particular engine. In the same figure the two different cycles traced by the cores run at SLS conditions are viewed (AEJK, ADLM).

It was hypothesized that a specific core compression ratio was more an indicator of the manufacturer's design philosophy (e.g. at what point the manufacturer divides the high and low pressure spools) than a critical design parameter. Figure 8 illustrates that the same mission can be accomplished with cores of differing compression ratios (but same maximum TIT) by simply adding the required temperature and pressure via compressor stages and or fans.

This brief analysis indicated that the core compression ratio, per se, is a somewhat arbitrary parameter, though measures of core performance such as pressure ratio/stage, and compressor efficiency were felt to be valid indicators of the competitive merit of a core.

C. FINITE BURNER RISE

Since gas generators are characterized by high values of TIT and relatively low compression ratios (8.0-14.0) certain testing procedures may require a combustor temperature differential above the core's capability, in order to reach maximum rated TIT. Reference 7 lists a maximum temperature differential of 2300° for an advanced combustor design, though the temperature rise across current core combustors is

more on the order of 1600° to 2000° (Reference 3). It can be seen in Figure 9 that a core with a compression ratio of 14.0 and rated at a TIT of $3000^{\circ}R$, but limited to a combustor rise of 1600° will fall short of the rated TIT. In investigating various test procedures this factor of burner deficiency must be considered.

IV. SELECTION OF PERFORMANCE PARAMETERS

To meaningfully describe the performance of a gas generator it was first necessary to carefully define a set of core performance parameters. Once defined these parameters will serve as the criteria for judging the merits of a core.

As a core is theoretically capable of being the integral part for any of the three basic gas turbine engines viz.; turbojet, turbofan, turboprop (turboshaft), performance parameters for these engines were analyzed with the thought that they might be easily modified so as to effectively characterize a core's performance.

As seen in Table I turbofans and turbojets are generally characterized by the amount of thrust produced, or some variation of this quantity e.g. F/m, F/A, F/W_e, and specific fuel consumption (SFC). Turboprop and turboshaft engines are rated in terms of thrust horsepower (THP) and SFC. Figure 10 traces out a typical thermodynamic cycle for a turbofan, turbojet, and turboprop (turboshaft) so that performance parameters for these various engines might be viewed in terms of their thermodynamic parameters.

Unfortunately engine parameters, as formulated, proved to be ill-defined when applied to a gas generator. For example, by-pass ratio has no significance when applied to a core. Though the engine parameters were not directly transferable to the gas generator, the philosophy of utilizing two parameters, one of which characterized energy available and the other fuel economy appeared to be valid concepts in describing a core's performance.

One parameter somewhat analogous to thrust, should describe the core's ability to produce useful energy. This energy could be utilized to provide thrust and/or power to drive the fans. The term gas horsepower (HP) was used to describe this available energy. To eliminate dependence on engine size, HP/m was used throughout this thesis rather than HP. Correspondingly a measure of the economics of providing this useful energy was desired (i.e. fuel consumed to produce this energy). Closely paralleling the SFC as described for a gas turbine engine, a SFC for the core was defined as the fuel consumed per unit of gas horsepower.

As discussed previously fluctuations in a gas generator's inlet (e.g. change in application) and outlet temperatures and pressures make performance, and therefore performance parameters, difficult to quantify and qualify. It was hoped to define gas horsepower and SFC in a manner that would circumvent this problem. Specifically it was desired that a gas horsepower could be formulated that would not only eliminate the influence of a particular application, but also the influence of core compression ratio. Conversely it was desired that the chosen parameters display a dependence on TIT and component efficiencies (η_c, η_b, η_t). As SFC is dependent on the definition of gas horsepower, final formulation of this parameter was held in abeyance until a workable definition of HP could be developed.

Three possible definitions of HP were postulated, each being analyzed in light of the above stated criteria (i.e. elimination of the influence of core compression ratio, inclusion of efficiencies etc.). Figures 11, 12 and 13 present a thermodynamic representation of each expression for HP for both a sea level static test of the core

and an "application" test (i.e. when the core is subjected to a temperature and pressure that simulates a particular application) since a testing procedure had not yet been determined. Inasmuch as one of the stated criteria for the formulation of HP was that it be independent of an application, those definitions of HP based on an application test were tentatively rejected.

A. DEFINITION "A"

Figure 11 describes the following definition of gas horsepower,

$$HP/m = C_p\ (T_{t4} - T_6) \tag{2}$$

This definition was initially appealing as it appeared that dependence on the compression ratio was precluded. Upon further analysis it was found that T_6 was a function of compression ratio as the following derivation shows.

The entropy rise across the combustor is given by;

$$\Delta S = \ln T_{t4}/T_{t3} - \frac{\gamma-1}{\gamma} \ln P_{t4}/P_{t3} \tag{3}$$

Assuming constant pressure across the combustor yields,

$$\Delta S = \ln T_{t4}/T_{t3} \tag{4}$$

For an isentropic compressor and turbine the entropy increase from $T_{t2.5}$ to T_6 along a constant pressure line equals that of expression (4). Thus,

$$\Delta S = \ln T_{t4}/T_{t3} = \ln T_6/T_{t2.5} \tag{5}$$

$$T_6 = T_{t4}/\tau_c \tag{6}$$

$$T_6 = (\pi_c^{(1-\gamma/\gamma)})T_{t4} \tag{7}$$

A more serious deficiency characterizing this method is the inability to reflect turbine inefficiencies. As previously shown compressor losses were seen to increase the value of T_6 (thus effectively reducing HP). Turbine losses are characterized on a Mollier diagram via pressure losses which cannot be accounted for directly in this definition. The possibility exists that this pressure loss can be converted to a lowered T_{t_4} or an increased value of T_6 via analytical manipulation. Such a scheme was not fully explored in this work.

B. DEFINITION "B"

Figure 12 depicts the following definition of gas horsepower,

$$HP/m = C_p \, (T_{t_{4.5}} - T_{t_{2.5}}) \qquad (8)$$

By equating T_6 to core inlet stagnation temperature, it was possible in some simple cases (e.g. isentropic compression) to eliminate the effect of compression ratio on HP. More realistic cases suffered, not only due to their dependence on compression ratio, but by the fact that, in a manner similar to definition "A", turbine losses were not accurately reflected in this definition of HP.

C. DEFINITION "C"

Figure 13 represents the following formulation for gas horsepower,

$$HP/m = C_p \, (T_{t_{4.5}} - T_6) \qquad (9)$$

This definition accurately reflects both compressor and turbine inefficiencies, though HP is still clearly a function of compression ratio. Assuming a satisfactory method of extrapolation could be

developed (see DISCUSSION AND RESULTS) to convert HP and SFC data for a core with a fixed compression ratio to HP and SFC values of a core with an increased compression ratio, the original dependence on core compression ratio could then be interpreted as dependence on the total compression ratio of either a turbojet or turbofan. This is analogous to building a turbojet or turbofan (less the by-pass air) out of the core, where HP (i.e. energy available) would be available for conversion to thrust for a turbojet or thrust and/or work to compress the by-passed air for a turbofan. This interpretation permits a viable comparison method to be established.

Once a definition of HP was established formulation of SFC was relatively straightforward. SFC was previously defined as the fuel consumed per unit of gas horsepower.

$$SFC = \frac{m_f}{HP}(3600) \tag{10}$$

where m_f equals fuel flow rate in units of lbm/sec.

Assuming that the mass flow rate of air is much greater than that of the fuel rate, 100% combustor efficiency and constant specific heat capacity (C_p) yields the fuel/air ratio (f),

$$f = \frac{m_f}{m_a} = \frac{C_p\,(T_{t_4} - T_{t_3})}{Q} \tag{11}$$

where Q equals fuel heating value. A value of Q for JP-4 is 18,000 BTU/lbm. Substituting equations (9) and (11) into (10) results in,

$$SFC = \frac{m_f/m_a}{HP/m_a} = \frac{T_{t_4} - T_{t_3}}{Q\,(T_{t_{4.5}} - T_6)} \tag{12}$$

V. CORE TESTING PROCEDURES

A. TEST CONDITIONS

In keeping with the stated goal of a simple test the following three alternatives were investigated to determine the most practical yet meaningful test conditions:

1. Reference Temperature and Pressure

As seen in Figure 14, the reference temperature and pressure method subjects the core to inlet temperature and pressure above that of standard sea level conditions. Essentially this is a simulation of an application where the reference temperature and pressure correspond to core inlet values in a particular application. In choosing core performance parameters, definitions dependent on an application were tentatively rejected. Figure 15, in conjunction with Table II, serves to further validate this decision. It is evident from Figure 15 that certain tests became impractical due to the very high pressure requirements. (e.g. high Mach number sea level static tests.) Table II and Figure 15 also provide temperature and pressure plots pertaining to the other suggested test methods.

2. Standard Temperature and Pressure

Figure 16 shows a sketch of an alternate test. Inlet temperature and pressure are standard sea level conditions, 519°R and 1.0 atmosphere. Such a test for most gas generators would result in a burner deficiency in that the core would fall short of its maximum TIT; see Figure 9.

3. Reference Temperature and Standard Pressure

As viewed in Figure 17 this test is conducted at an augmented temperature and at a pressure of one atmosphere. The reference temperature,

rather than being associated with a particular application is that temperature required for core operation at maximum rated TIT and 100% corrected RPM. See V. C.1, Determination of Input Conditions. By testing at an increased temperature one derives the additional benefit of subjecting the core to realistic thermal stresses. Since the pressure is maintained at one atmosphere and incoming air is simply pre-heated, the test satisfies the criterion of simplicity.

One further test could be postulated, i.e. "Reference Pressure and Standard Temperature." This method was not seriously considered as it combined both the undesirable features of Tests 1 and 2.

B. SIMILARITY

Reference 2 describes the requirements for, and the benefits of similarity testing for gas turbine engines. Investigation revealed that such requirements and benefits were directly transferable, with minor modifications, to gas generator testing.

In a gas generator, like a gas turbine engine, maintenance of similarity requires that the Mach number (M), Reynolds number (R_e) and dimensionless RPM (NL/a) be fixed. A practical consequence of similarity testing is the fact that any geometrically similar gas generator (e.g. the same core) operating at the same M, R_e and dimensionless RPM will have the same values for dimensionless performance parameters and eliminate the influence of both altitude and scale.

In this brief discussion of similarity, parameters have been specified as dimensionless. In actual testing it is often desired to re-insert dimensions into the parameter, not only permitting

identification with a particular variable, but also to bring the numerical values of these parameters in consonance with actual values.

One method of accomplishing this task is the introduction of corrected quantities. Corrected parameters are directly proportional to their dimensionless counterparts. For example, as shown in Reference 2, corrected RPM ($N/\sqrt{\theta}$) is proportional to dimensionless RPM[NL/a]. Corrected core performance parameters were found to be $\frac{HP}{\sqrt{\theta}\,\delta}$ and SFC. Measured SFC is the same as corrected SFC.

By definition, θ is the ratio of T/T_{STD}, thus necessitating that both T and T_{STD} be specified. Several choices were considered:

$$T_{STD} = T_{t_{REF}} \text{ (reference total temperature)} \tag{13}$$

$$T_{STD} = T_{REF} \text{ (reference static temperature)} \tag{14}$$

$$T_{STD} = T_{SEA\ LEVEL} \text{ (standard day)} \tag{15}$$

It was felt that T_{STD} should be equated to the standard day sea level temperature (519°R) as this would establish a reference point common to all gas generators. T was defined as the reference total temperature.

δ is the ratio of P/P_{STD}. Somewhat analogous to the above temperature deliberations P_{STD} was set at a pressure of one atmosphere. P was defined as the actual static inlet pressure.

C. ACTUAL TEST

The actual gas generator test requires that the air be preheated to an augmented temperature in order that the maximum TIT is obtained. Measurements to be taken must be stipulated, and finally a data reduction scheme must be specified.

1. <u>Determination of Inlet Conditions (Reference Temperature)</u>

In determining a reference temperature for a particular gas generator, certain preliminary data and calculation are required. Specifically it is necessary to know the value of the maximum rated TIT at a definite core operating condition. This information would normally be specified by the manufacturer. The temperature ratio across the combustor (τ_b) at 100% corrected RPM is needed; for this case,

$$\theta = T_{ambient}/T_{STD} \tag{16}$$

If the combustor ratio (τ_b) is not provided such a value could be determined by conducting the "cold test" described below.

The "cold test" refers to the fact that the core inlet temperature is the standard temperature rather than an augmented value (see figure 16). The core is tested at sea level static conditions and 100% corrected RPM. A value of T_{t_4} is measured at these conditions. A standard compressor map provided by the manufacturer is entered, and at the intersection of the design line and the 100% corrected RPM line a compression ratio and compressor efficiency are defined. See Figure 18. As shown in reference 6, the compressor temperature ratio can be calculated as follows,

$$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \tag{17}$$

Thus the "cold test" provided two important ratios viz.; τ_c and τ_b, Since, by definition, a similarity test guarantees all temperature ratios will be maintained, the following relationships hold:

$$\tau_{b_{COLD}} = \tau_{b_{HOT}} \qquad (18)$$

$$\tau_{c_{COLD}} = \tau_{c_{HOT}} \qquad (19)$$

where the subscript HOT refers to the fact that the inlet air has been preheated.

The desired quantity, $T_{t_{2.5}}$, can now be easily determined. Combining equation (18) and the definition of τ_b,

$$T_{t3_{HOT}} = T_{t4_{MAX}} / \tau_{b_{COLD}} \qquad (20)$$

Equation (19) and the definition of τ_c yields,

$$T_{t2.5_{HOT}} = T_{t3_{HOT}} / \tau_{c_{COLD}} \qquad (21)$$

As illustrated in Figure 17, the test is then conducted under the following conditions:

$$T_{t_2} = T_{t2_{HOT}}$$

$$P_{t_2} = P_{t_o} = P_o$$

$$N/\sqrt{\theta} = 100\%$$

$$\theta = T_{t2_{HOT}} / T_{SEA\ LEVEL}$$

It can be seen that preheating makes θ greater than one. This, in turn, requires that the actual RPM (N) be greater than the corrected RPM ($N/\sqrt{\theta}$). For example, let $N/\sqrt{\theta}$ equal a constant value of 10,000 RPM. If the ambient temperatures equalled 519°R then N for a "cold test" would be exactly equal to 10,000 RPM. The actual N for the hot test would be some value above 10,000, say 11,000 RPM.

2. Measurements and Data Reduction

No attempt will be made in this thesis to describe the exact instrumentation required to measure the desired quantities. All measurements required were felt to be routine.

If a compressor map is provided, the "cold test" simply requires that T_{t_4} be measured. As the hot test is the gas generator performance test, quantities must be measured that permit the computation of gas horsepower and SFC. Recalling equations (9) and (10) it is evident that the following parameters are required:

$\dot{m}_a$ = mass flow air rate (lbm/sec)

m_f = mass flow fuel rate (lbm/sec)

$T_{t_{4.5}}$ = total turbine outlet temperature

T_6 = static exhaust temperature

Location of temperature and pressure subscripts are shown in figure 17. It is difficult, if not impossible, to accurately measure static temperature (T_6) in the exhaust stream. The computational scheme developed to determine T_6 will now be discussed. Assuming a calorically and thermally perfect gas, the equation of the curves on the Mollier diagram are given by,

$$\frac{S - S'}{C_p} = \ln \frac{T}{T'} - \frac{\gamma-1}{\gamma} \ln \frac{p}{p'} \tag{22}$$

Thus the entropy increase from point 2.5 to 4.5 is expressed by,

$$\frac{\Delta S}{C_p} = \ln \frac{T_{t_{4.5}}}{T_{t_{2.5}}} - \frac{\gamma-1}{\gamma} \ln \frac{P_{t_{4.5}}}{P_{t_{2.5}}} \tag{23}$$

Assuming $P_6 = P_0$ and $T_{2.5} = T_{t_{2.5}}$, the entropy rise from 2.5 to 6 is,

$$\frac{\Delta S}{C_p} = \ln \frac{T_6}{T_{t_{2.5}}} \tag{24}$$

If an isentropic expansion is assumed from 4.5 to 6 equations (23) and (24) can be equated yielding,

$$T_6 = T_{t_{4.5}} - T_{t_{2.5}} \left(\frac{P_{t_{4.5}}}{P_{t_{2.5}}}\right)^{\frac{\gamma-1}{\gamma}} \tag{25}$$

Equation (25) requires a measurement of $T_{t_{2.5}}$, $P_{t_{4.5}}$, $P_{t_{2.5}} = P_o$.

Values of $T_{t_{2.5}}$ and $P_{t_{2.5}}$ are also needed in the computation of θ and δ. With a value of T_6 established and knowledge of the other specified parameters HP and SFC can be computed from equations (9) and (10).

VI. CRITERIA FOR CORE COMPARISON

A. PROBLEM STATEMENT

Once the performance parameters, HP/m and SFC, were formulated and a testing procedure established there remained the question, "How are test data meaningfully compared so as to identify the best core?" An alternate statement of the problem was: given two cores "A" and "B" and a multiplicity of missions, identified by I, II, III, IV, etc., derive a simple comparison criterion that will evaluate AI vs BI (i.e. core A used in an engine optimized for mission I and core B used in an engine optimized for mission I), AII vs BII, AIII vs BIII, etc.

When compared, cores fall into two general categories. Category 1 is defined as follows,

$$\text{MAX TIT}_A > \text{MAX TIT}_B$$

$$\eta_{cA} \geq \eta_{cB}$$

$$\eta_{bA} \geq \eta_{bB}$$

$$\eta_{tA} \geq \eta_{tB}$$

Figure 19 shows a SLS plot of HP/m vs SFC of two 100% efficient cores--"A" and "B". Throughout this section core "A" will always be rated at 3000°R for TIT while core "B" will be rated at 2500°R. The plot of Figure 19 was generated by allowing core compression ratios to vary. This is somewhat analogous to changing total compression in an engine to accomodate a particular mission. As evidenced by Figure 19, regardless of the mission specified core "A" will always have the capability to out perform "B". Capability is stressed, as a non-judicious (e.g. not optimum) choice of input conditions (temperature

and pressure) might permit "B" to surpass "A" in performance. Though curves result from a SLS test, comparisons may be made over a wide spectrum of conditions. For example, at Mach 1.5 and at an altitude of 30,000 feet it is possible to predict that an engine built from core "A" will possess the capability to out perform an engine using "B". In summary, the core with the highest TIT and with all component efficiencies equal or greater than the lower TIT core, will always be capable of yielding better performance than the lower TIT core. Thus cores that fall into Category 1 are seen to be readily amenable to comparison.

Category 2 is specified as follows,

$$\text{MAX TIT}_A \geq \text{MAX TIT}_B$$

$$\eta_{iA} \leq \eta_{iB}$$

where subscript i refers to compressor, burner or turbine efficiencies.

Even though "A" is rated at a higher TIT than "B", lower efficiencies in one or more of its components preclude an intuitive comparison of "A" vs "B".

B. METHOD OF COMPARISON

Three methods of comparison were investigated for the Category 2 cores. The first two will be described only briefly as they suffer various deficiencies that rendered them unsuitable for comparison.

A series of HP/m vs SFC curves were generated (via an extrapolation of compression ratios) as shown in Figure 20. The major drawback to this method is that two parameters, HP/m and SFC, must be "traded off" dependent on the mission specified. This theoretically allows an infinite number of points to be considered and reviewed.

A second method computed actual values of HP/m and SFC for cores "A" and "B" and formed the ratios HP_{ACTUAL}/HP_{IDEAL} and SFC_{ACTUAL}/SFC_{IDEAL}. The measure of core excellence was theorized to be the divergence, or more accurately the lack of, from the ideal case. This method appeared to possess a certain degree of validity as core "B" was chosen in such a manner to be visibly inferior to "A". Core "B" clearly displayed a much more dramatic divergence from the ideal case than did "A". Unfortunately no technique was found that permitted conversion of this divergence into a viable comparison criteria.

Having witnessed the shortcomings of the aforementioned methods a criteria was sought that would characterize the merit of the core via a single parameter. Previously it was shown (Figure 19) that if two cores were 100% efficient the core with the highest TIT would exhibit the capability for better performance than the lower TIT core. Combining these two thoughts (i.e. single parameter and higher TIT) led to the formulation of a core merit factor -- TIT Effective (TIT_E). TIT_E is the result of reducing a core with a specified TIT and losses into a 100% efficient core with a lowered value of TIT (i.e. TIT_E) to reflect this adjustment.

Solving for TIT_E requires that an expression for actual gas horsepower and SFC be developed. The necessary formulas, derived in Appendix A, are:

$$\frac{HP}{m} = C_p\, T_{t_0} \left\{ \tau^* \left[1 - \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c\, \tau^*} \right] \left[1 - \frac{1}{(\pi_c \pi_b)^{\frac{\gamma-1}{\gamma}} \left(1 - \frac{\pi_c^{\frac{\gamma-1}{\gamma}}}{\tau^* \eta_c \eta_t} \right)} \right] \right\} \quad (26)$$

$$SFC = \left(\frac{3600\ C_p}{Q}\right)\left[\tau^* - 1 + \frac{(\pi_c^{\frac{\gamma-1}{\gamma}} - 1)}{\eta_c}\right] \tag{27}$$

Actual values of HP/m and SFC (Equations (26) and (27)) are set equal to ideal values (Equations (9) and (10)). The resulting simultaneous equations are solved for the quantity — TIT_E. An explanation of the computer program to calculate TIT_E is found in Appendix B.

The core test as specified in section V will yield only a single value of HP/m and SFC. These values are converted into a value of TIT_E via the computational technique just discussed. Figure 21 depicts this result for cores "A" and "B". A single point for each core must be extrapolated into a curve which is a function of compression ratio. Figure 22 shows the result of one such extrapolation when it was assumed that $\eta_c = \eta_t$ and $\eta_b = 1.0$ and that component efficiencies could be maintained as the compression ratio increased. (See <u>RESULTS AND DISCUSSION</u> for further comments on the extrapolation process.)

In order to simplify the ensuing discussion on the use of TIT_E it was helpful to define the following compression ratios:

π_{CA} = pressure ratio of core "A"

π_{CB} = pressure ratio of core "B"

π_C' = critical pressure ratio; value of π_C where curves cross

π_{CAI} = optimum pressure ratio for engine for mission I using core "A"

π_{CBII} = optimum pressure ratio for engine for mission II using core "B"

Figure 22 illustrates the above defintions.

The TIT_E concept can best be demonstrated via the use of Figure 23. Recalling that the goal of the TIT_E merit factor was to predict the

superiority of one core over another in specified mission(s), core "A" at 75% efficiency is compared with "B" at 100% efficiency. In this section X% efficiency refers to the fact that both turbine and compressor are X% efficient; combustor efficiency was set equal to unity. From Figure 23 it can be seen that the critical pressure ratio ($\pi_C{}'$) occurs at 14.0, point C. This suggests that core "A" will always yield better performance than "B" whenever the optimum mission pressure ratio for engine "A" is less than 14.0. Engine optimization for a particular mission will be reflected in a unique compression ratio for each core. Conversely, core "B" will be the better core when the optimum mission compression ratio is greater than 14.0.

As another example, core "A" at 85% efficiency according to the TIT_E theory will provide superior performance compared to core "B" with 100% efficiency in all missions which require an optimum compression ratio of less than 35.0. See point D on Figure 23. For optimum pressure ratios greater than 35.0, core "B" will always yield better performance regardless of the mission. These comparisons based on TIT_E will be further scrutinized in the RESULTS AND DISCUSSION section.

VII. ANALYTICAL MODEL

To test the suitability of the TIT_E concept it was necessary to fabricate a model that would optimize a given engine for a particular mission. As previously noted engine performance is usually stated in terms of F/m and SFC. To coincide with the TIT_E philosophy a single parameter, minimum propulsion weight, was chosen to characterize the optimum engine.

Propulsion weight for a turbojet was defined as the sum of the engine weight plus fuel weight required for the mission. An analagous definition for a turbofan is developed in Appendix C. Fuel weight can be expressed as,

$$\text{FUEL WT.} = (\text{A/C WT})\ (\text{L/D})^{-1}\ (\text{DURATION})\ (\text{SFC}) \qquad (28)$$

To determine an expression for engine weight it was necessary to separate the engine into three distinct sections viz., the inlet (diffuser), nozzle and/or afterburner, and the remainder—the engine. The weight of each of these sections was expressed as a product of the mass flow rate through the engine (m_a) and a constant (c_i).

$$\text{weight of inlet} = c_1 m_a$$

$$\text{weight of nozzle} = c_2 m_a$$

$$\text{weight of engine} = c_3 m_a$$

It was found that these constants varied with the engine type. Typical values of c_i for a high Mach number turbojet were,

$$c_1 = \frac{3 \text{ lbs}}{(\dot{m})}$$

$$c_2 = \frac{3 \text{ lbs}}{(\dot{m})}$$

$$c_3 = \frac{20 \text{ lbs}}{(m)}$$

Values for these constants were determined by averaging pertinent data from high Mach number engines. Data were obtained from Reference 5.

Collecting terms yielded the following expression for propulsion weight,

$$\text{PROP WT} = \frac{(\text{A/C WT})}{(\text{L/D})} \left\{ [c_1 + c_2 + c_3] \frac{m}{F} + (\text{SFC})\ (\text{DURATION}) \right\} \tag{29}$$

To utilize equation (29) values for F/m and SFC are required. Although the propulsion weight model was only exercised for a turbojet, expressions for F/m and SFC were developed for the more general case of turbofan with losses. A specific case, like a 100% efficient turbojet, could easily be handled by setting the by-pass ratio (β) to zero and setting all efficiencies to unity. The derivation of the following formulas for F/m and SFC, applicable to a turbofan with losses, closely parallels the turbojet derivation of Reference 6 for the same parameters. Refer to Figure 1 for subscript locations.

$$\tau_c = 1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \tag{30}$$

$$\tau_t = 1.0 - \tau_r \left\{ \frac{(\tau_c - 1) + \beta(\tau_c' - 1)}{\tau^*} \right\} \tag{31}$$

$$U_{90} = \sqrt{\frac{\tau_r \tau_c' - 1}{\tau_r - 1}} \tag{32}$$

$$U_{80} = \sqrt{\frac{\tau^* \tau_t}{\tau_r(\tau_r - 1)} \left(\tau_r - \frac{1}{1 + \eta_c(\tau_c - 1)(1 - \frac{1-\tau_t}{\eta_t})} \right)} \tag{33}$$

$$\frac{F}{m\, g\left(\frac{1+\beta}{\beta}\right)} = \frac{a_o\, M_o}{g} \left\{ \left(\frac{1}{1+\beta}\right) U_{80} + \left(\frac{\beta}{1+\beta}\right) U_{90} - 1 \right\} \tag{34}$$

$$SFC = \left(\frac{3600\ C_p\ T_{t0}}{Q}\right) \left\{ \frac{\tau^* - \tau_r \left(\frac{1+\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \right)}{B(F/mg)} \right\} \tag{35}$$

A computer program was developed that provided the capability to calculate the optimum engine for a specified mission via the minimum propulsion weight criteria. Explanation of this program is found in Appendix D.

VIII. DISCUSSION AND RESULTS

To test the validity of the TIT_E concept two missions were chosen. Both missions were flown at an altitude of 30,000 feet at Mach 1.5. Mission I was of .3 hours duration while mission II was 1.5 hours. In each mission propulsion weight was minimized for the two model engines, "A" and "B". These engines contained cores "A" and "B", thus engines were rated at the TITs specified for the cores. ("A" = 3,000°R, "B" = 2500°R). Table III presents a summary of the results obtained from computer program II. For each mission and specified efficiency, the resulting minimum propulsion weight and the corresponding compression ratio are tabulated. Efficiency refers to the value of the compressor or turbine efficiency which were set equal for computational ease.

It was postulated that when the two cores being compared were determined to be in Category 1, the core with the highest value of TIT would be superior to the lower valued TIT core for all missions. Table IV presents a matrix of the possible comparison categories between "A" and "B". Table III indicates the validity of this theory as, for all cases, the core with the highest TIT, i.e. "A", produces the minimum propulsion weight.

If the TIT_E concept is to prove valid the predicted engine performance based on TIT_E must correlate with actual engine performance. Two previously discussed comparisons involved "A" at 85% efficiency vs. "B" with 100% efficiency. As seen from Figure 23, the TIT_E merit factor predicted that core "A" would exhibit better performance than "B" when the optimum mission compression ratio was less than the critical pressure ratio. Entering Table III it is seen that π_{CAI} is less than

$\pi_C{}'$. As predicted for mission I, an engine containing core "A" will demonstrate superior performance, in terms of propulsion weight, compared to an engine utilizing core "B". For mission II, Table III indicates that π_{CAII} is greater than $\pi_C{}'$ and predictions based on TIT_E are seen to be valid. Using Figure 23 and Table III the validity of the TIT_E ccncept can easily be corroborated for the comparison of "A" at 75% efficiency and "B" at 100% efficiency for both missions I and II.

The remaining Category 2 comparison considers "A" at 75% efficiency vs. "B" at 85% efficiency. As seen in Figure 24 two limit points are defined; the critical pressure ratio (point E) and a lower limit (point F). Point F is a critical point since core "B" at 85% efficiency can never exceed the TIT_E for "B" with 100% efficiency (2500^{o}R). The intersection cf the 2500^oTIT_E line with the "A" 75% line defines the lower limit, point F. To the left of this point, π_C = 14.0, core "A" would always exhibit better performance than core "B". Interpretation of the remaining two zones in Figure 24, i.e. where π_C is between 14.0 and the critical pressure ratio and where π_C is greater than the critical pressure ratio, yields the following comparisons. Where π_C is between 14.0 and the critical pressure ratio, either "A" or "B" could provide better performance dependent on the mission. If it is known that a mission employing engine "A" requires a compression ratio of 48.0 and the same mission utilizing "B" requires a pressure ratio of 32.0, Figure 24 indicates that core "B" has the higher TIT_E and therefore will be superior to "A". In the region where π_C is greater than the critical pressure core "B" will provide the better performance.

The ambiguity of this region was solved by noting that for any mission tested the engine with the lower value of TIT always optimized at a lower compression ratio. Thus it can be predicted that for any mission requiring an optimum pressure ratio greater than π_C', an engine built from core "B" will be superior to an engine utilizing "A". Figure 24, in addition to the above example, demonstrates the validity of the TIT_E merit factor by comparing optimum engine pressure ratios for various Category 2 efficiencies and missions. In all cases the core TIT_E merit factor correctly predicted the best engine.

As previously discussed the core test yields only one value of HP/m and SFC. From this set of values a TIT_E can be computed. To make meaningful comparisons it is necessary to extrapolate this one data point into a curve of TIT_E vs. compression ratio. The analytical model in this thesis required a major assumption to accomplish this extrapolation. It was assumed that the given compressor and turbine efficiencies could be maintained as the compression ratio increased. It should also be noted that Figures 23 and 24 represent TIT_E vs π_C for these cases where compressor efficiency equals turbine efficiency. Where these efficiencies differ curves would assume different slopes. Though this method of extrapolation appeared valid for analytical investigation, problems might arise in extrapolating real data.

All comparisions were based on a turbojet model. It was felt that the TIT_E concept with modifications would be capable of making valuable predictions concerning gas generators to be used in turbofans or turboprop engines. Unfortunately time did not permit the necessary investigation.

All analytical tests and comparisons for both the core and the engine were conducted at 100% RPM. This was done to ease computation and to avoid dependence on difficult-to-obtain data. It is felt that all proposed tests, models and comparisons will remain valid at engine speeds less than 100%.

IX. CONCLUSIONS

No single core merit factor was found that clearly delineated the best core for all missions. This results from the fact that core performance, as expressed by any meaningful set of parameters, is application dependent.

TIT_E was judged to be a valid merit factor that permitted core comparisons over a broad range of applications. For some missions, cores could only be compared in the limiting case unless a mission in terms of π_c was known.

Further investigation is required to determine the most valid method of extrapolating data from the core test into curves of TIT_E vs compression ratio. Specifically, this is needed when compressor, turbine and combustor efficiencies are significantly different from each other.

Further study is also needed to determine the applicability of the TIT_E concept to gas turbine engines other than turbojets, viz., turbofans and turboprops.

TABLE I

GAS TURBINE PERFORMANCE PARAMETERS

ENGINE	PARAMETER	FORMULA
Turbojet	F/m	$F/m = C_1^{+}\left\{\sqrt{\frac{1}{\tau_r - 1}\left[\tau^{*}\left(1 - \frac{1}{\tau_r \tau_c}\right) - \tau_r(\tau_c - 1)\right]} - 1\right\}$
	SFC	$SFC = \frac{3600\ \tau_c\ (\tau_b - 1)}{(Q/C_p\ T_{t0})(F/m)}$
Turbofan	F/m_t	$F/m_t = C_1^{+}\left\{\left(\frac{1}{1+B}\right) U_{80}^{++} + \left(\frac{B}{1+B}\right) U_{90}^{+++} - 1\right\}$
	SFC	$SFC = \left(\frac{3600}{Q/C_p\ T_{t0}}\right) \frac{\tau^{*} - \tau_r\ \tau_c}{B(1 + 1/B)\ F/m_t}$
Turboprop	THP	$THP = SHP^{\$} + \frac{m}{g}\left[V_{ef}\ (1 + f - V_0)\right] \frac{V_0}{550}$
	SFC	$SFC = \frac{m_f}{THP}$

$+ \quad C_1 = a_o\ M_o/g$

$$++ \quad U_{80} = \sqrt{\frac{\tau^{*}}{\tau_r - 1}\left\{1 - \frac{\tau_r}{\tau^{*}}\left[(\tau_c - 1) + \beta(\tau_c' - 1)\right] - \frac{1}{\tau_r \tau_b}\right\}}$$

$$+++ \quad U_{90} = \sqrt{\frac{\tau_r \tau_c' - 1}{\tau_r - 1}}$$

\$ SHP = SHAFT HORSEPOWER DELIVERED TO GEARBOX

TABLE II

CORE TEMPERATURE AND PRESSURE
REQUIRED FOR TESTING

TEST	REF T & P	STD T & P	REF T
T_t [^oR]	697	519	697
P_t[ATMS]	2.8	1.0	1.0

TEMPERATURE AND PRESSURE FOR STATIC TEST
CORE WITH π_c = 10.0, AND ENGINE π_c = 28.0

ALTITUDE \ MACH RANGE		.5 → .7	1.0 → 1.2	1.5 → 2.2
SEA LEVEL	T_t[^oR]	731-764	836-897	1009-1370
	P_t[ATM]	3.32-3.88	5.3-6.8	10.27-29.9
36,000 FEET	T_t[^oR]	549-574	628-674	758-1039
	P_t[ATM]	.75-.874	1.19-1.53	2.31-6.73

TABLE III

OPTIMUM PROPULSION WEIGHT VS
COMPRESSION RATIO

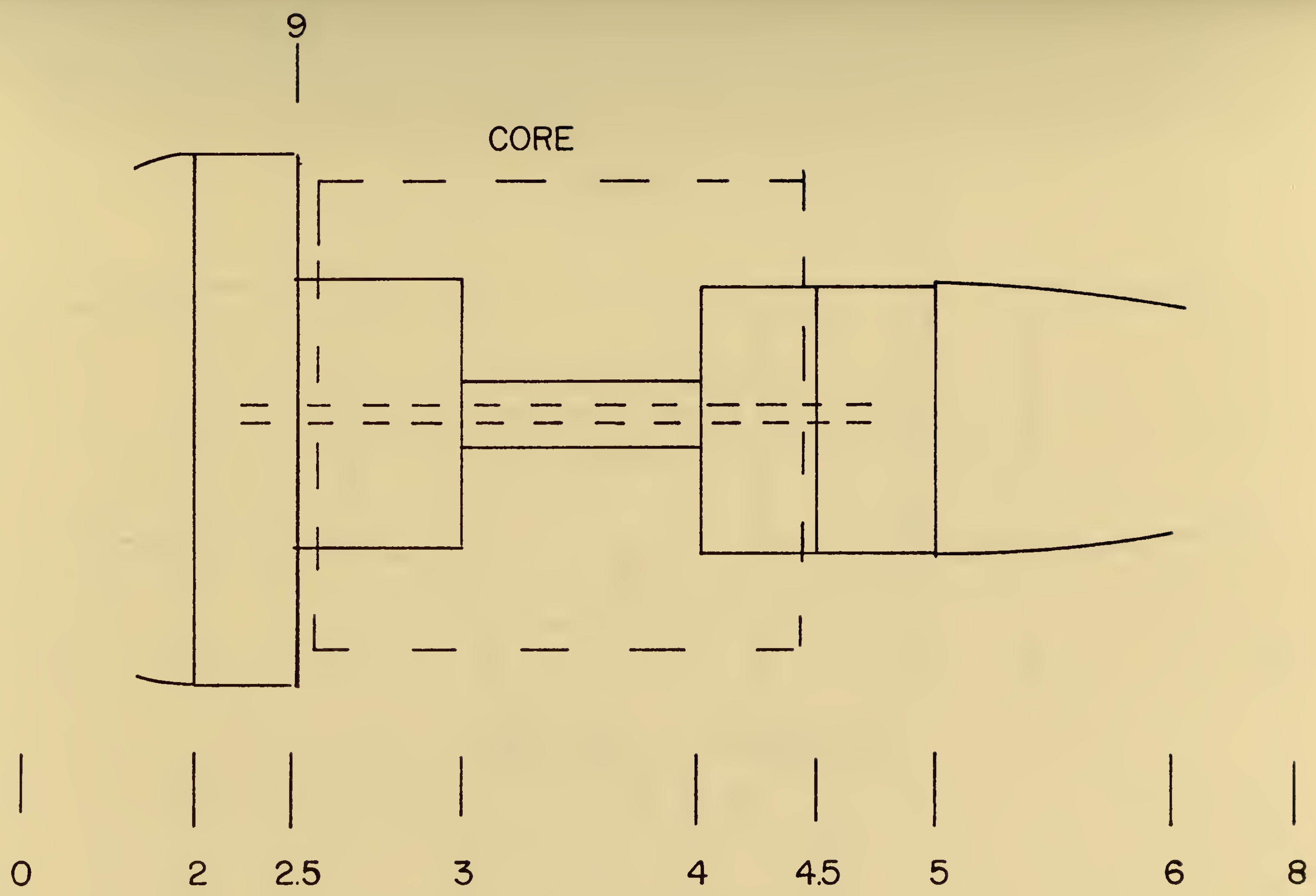
	.3 HR, MISSION I		1.5 HR, MISSION II	
	A	B	A	B
$\eta_c = \eta_t$	1.0	1.0	1.0	1.0
π_c OPT	30.0	16.0	> 62.0	54.0
PROP. WT.	3,778	4,179	9,769	9,934
$\eta_c = \eta_t$	.85	.85	.85	.85
π_c OPT	28.0	16.0	> 62.0	32.0
PROP WT	4,161	4,664	11,400	11,985
$\eta_c = \eta_t$	.75	.75	.75	.75
π_c OPT	24.0	14.0	48.0	22.0
PROP WT	4,558	5,152	13,099	13,890

TABLE IV

MATRIX OF POSSIBLE COMPARISON CATEGORIES

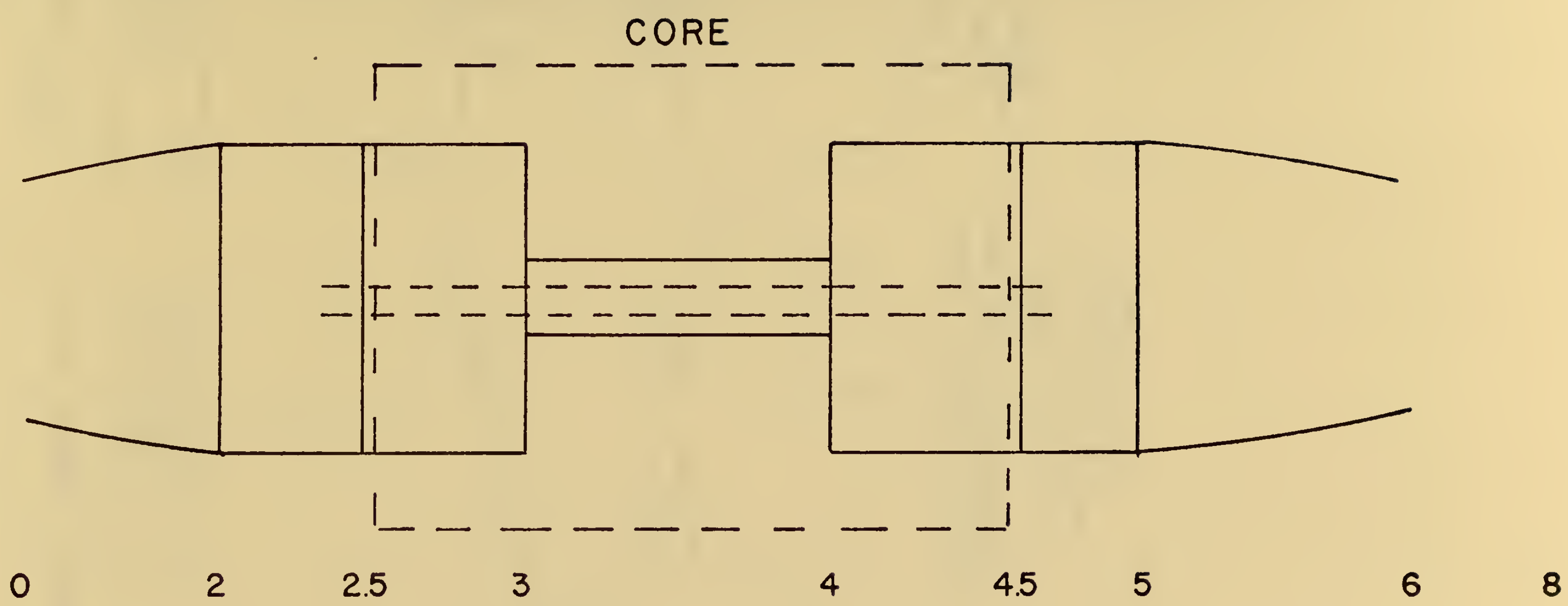
BETWEEN CORES "A" AND "B"

		CORE "A"		
		75%	85%	100%
CORE "B"	75%	CATEGORY "1"	CATEGORY "1"	CATEGORY "1"
	85%	CATEGORY "2"	CATEGORY "1"	CATEGORY "1"
	100%	CATEGORY "2"	CATEGORY "2"	CATEGORY "1"



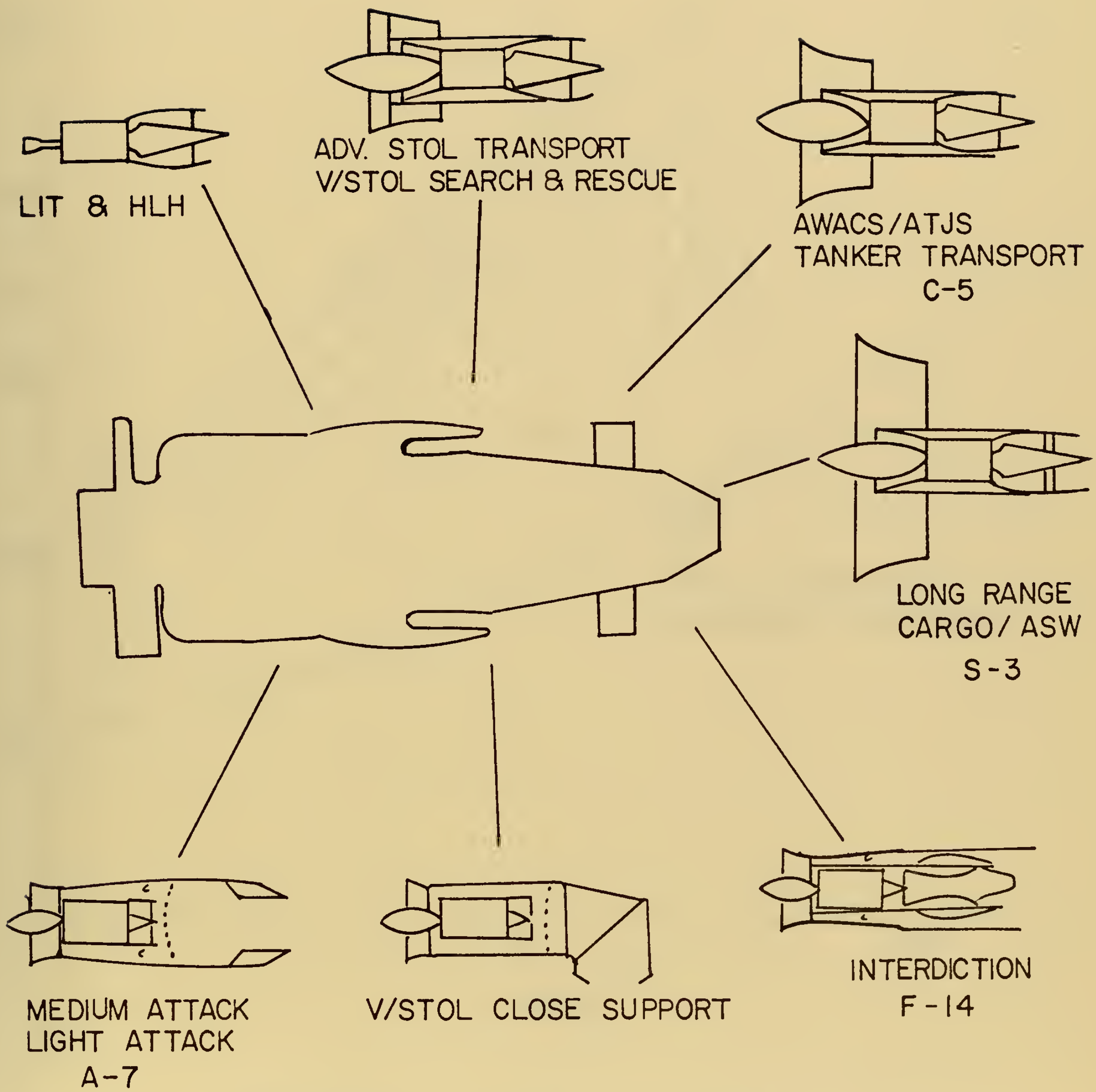
CORE OF A TURBOFAN

FIG. 1



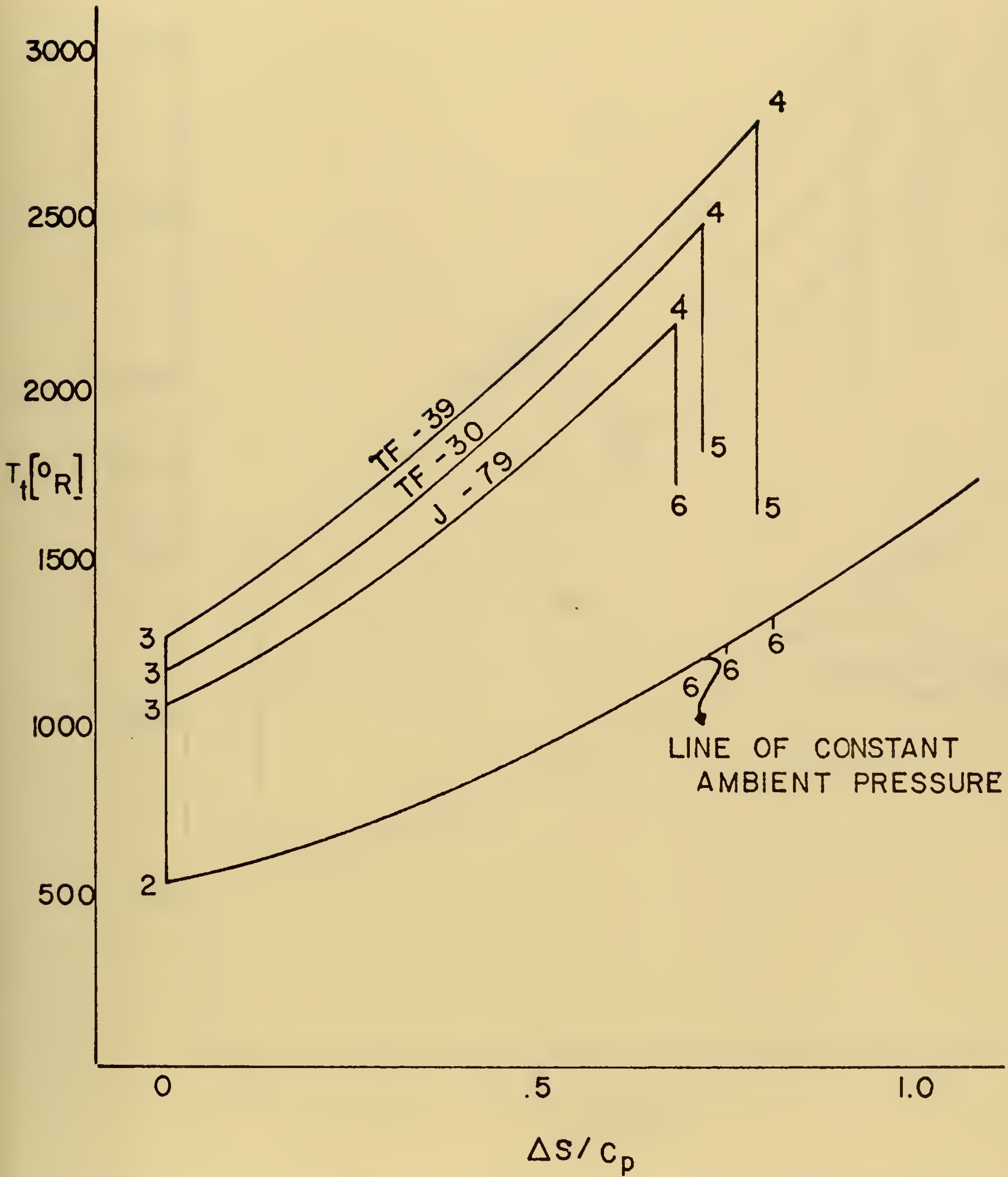
CORE OF A TURBOJET

FIG. 2



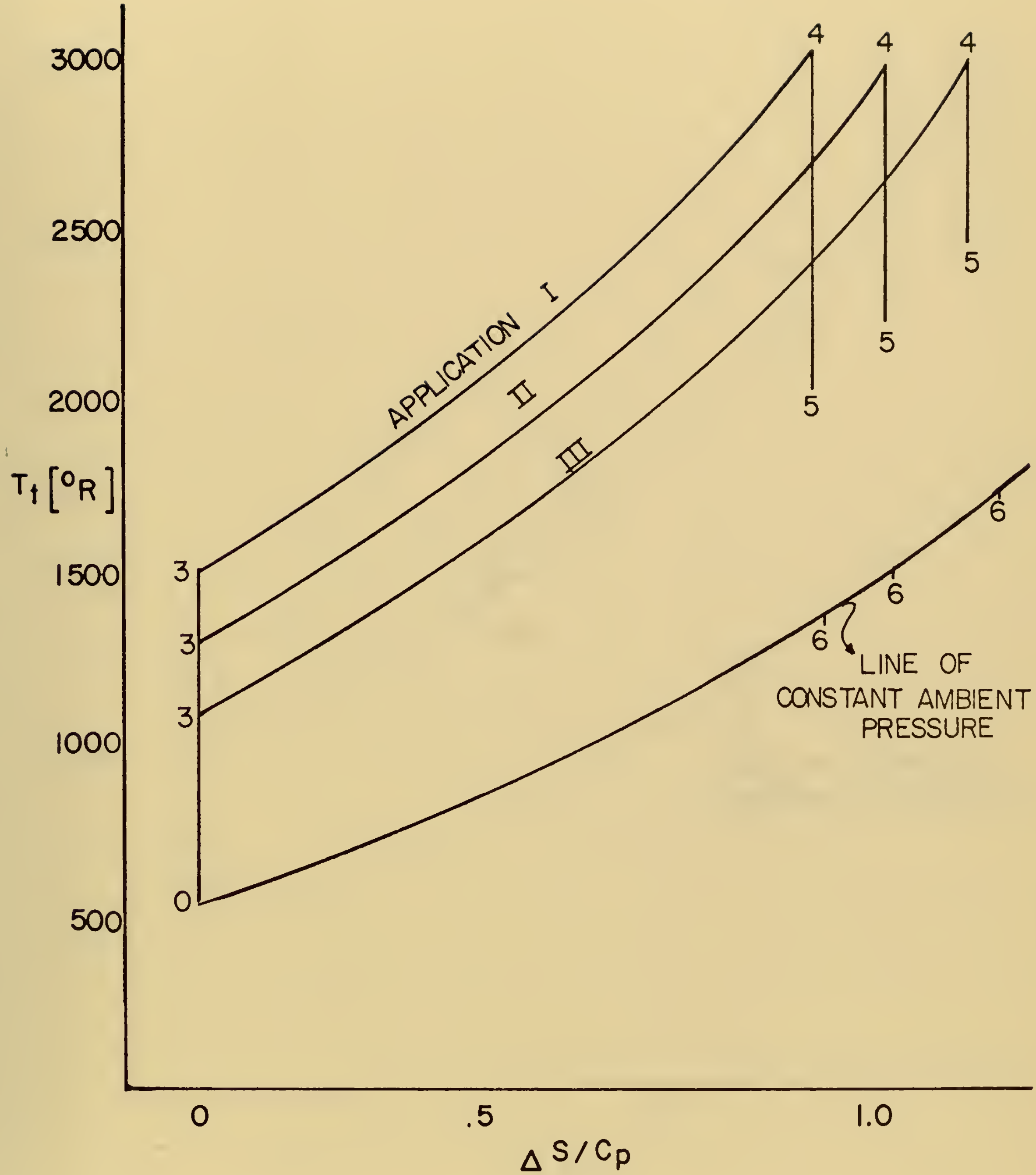
ENGINES DERIVED FROM
A CORE

FIG. 3



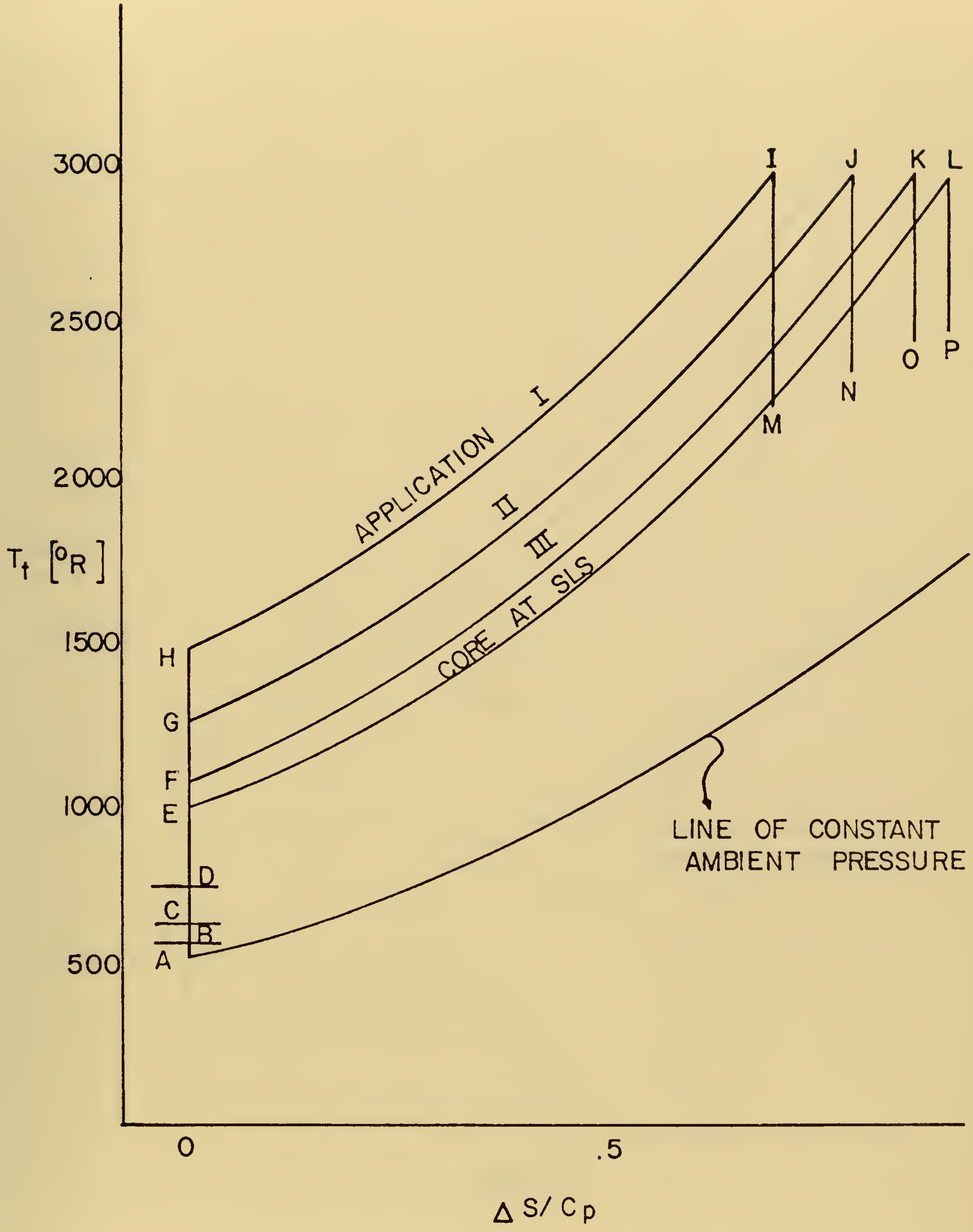
SEA LEVEL CYCLES TRACED BY
THREE DIVERSE ENGINES

FIG. 4



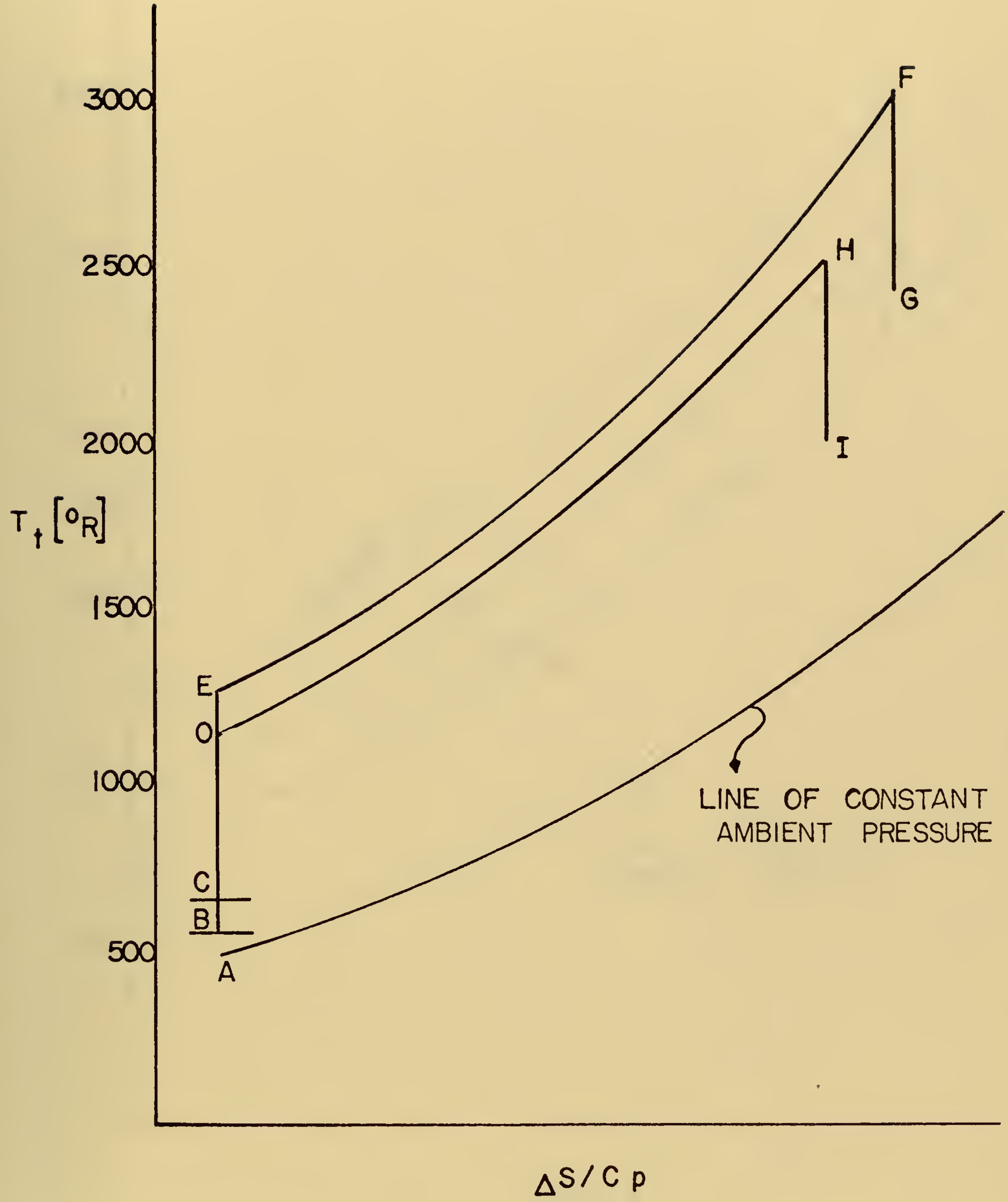
SEA LEVEL STATIC CYCLES TRACED BY
ENGINES UTILIZED IN THREE APPLICATIONS

FIG. 5



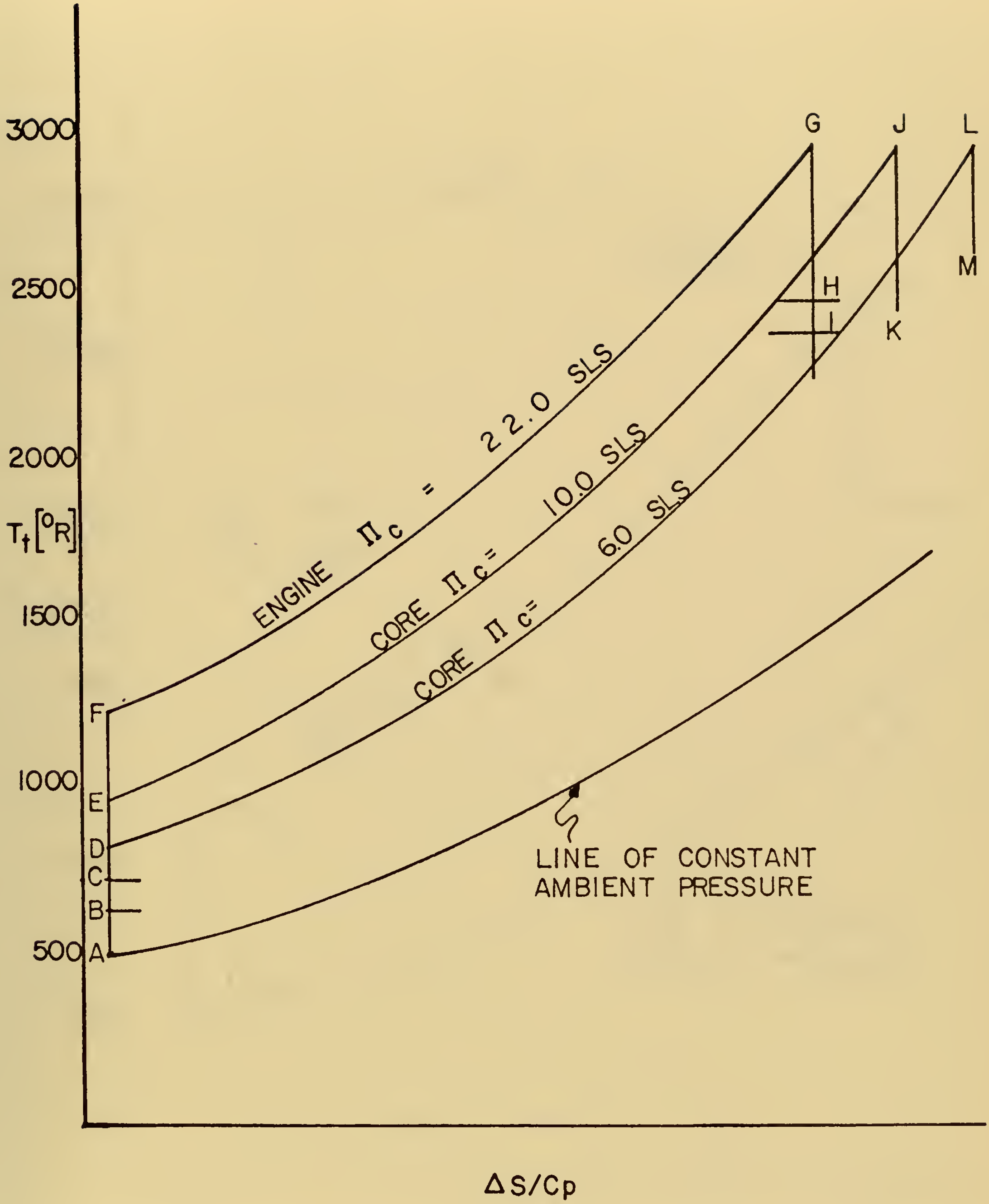
CORE APPLICATION VS. CORE TEST STAND

FIG. 6



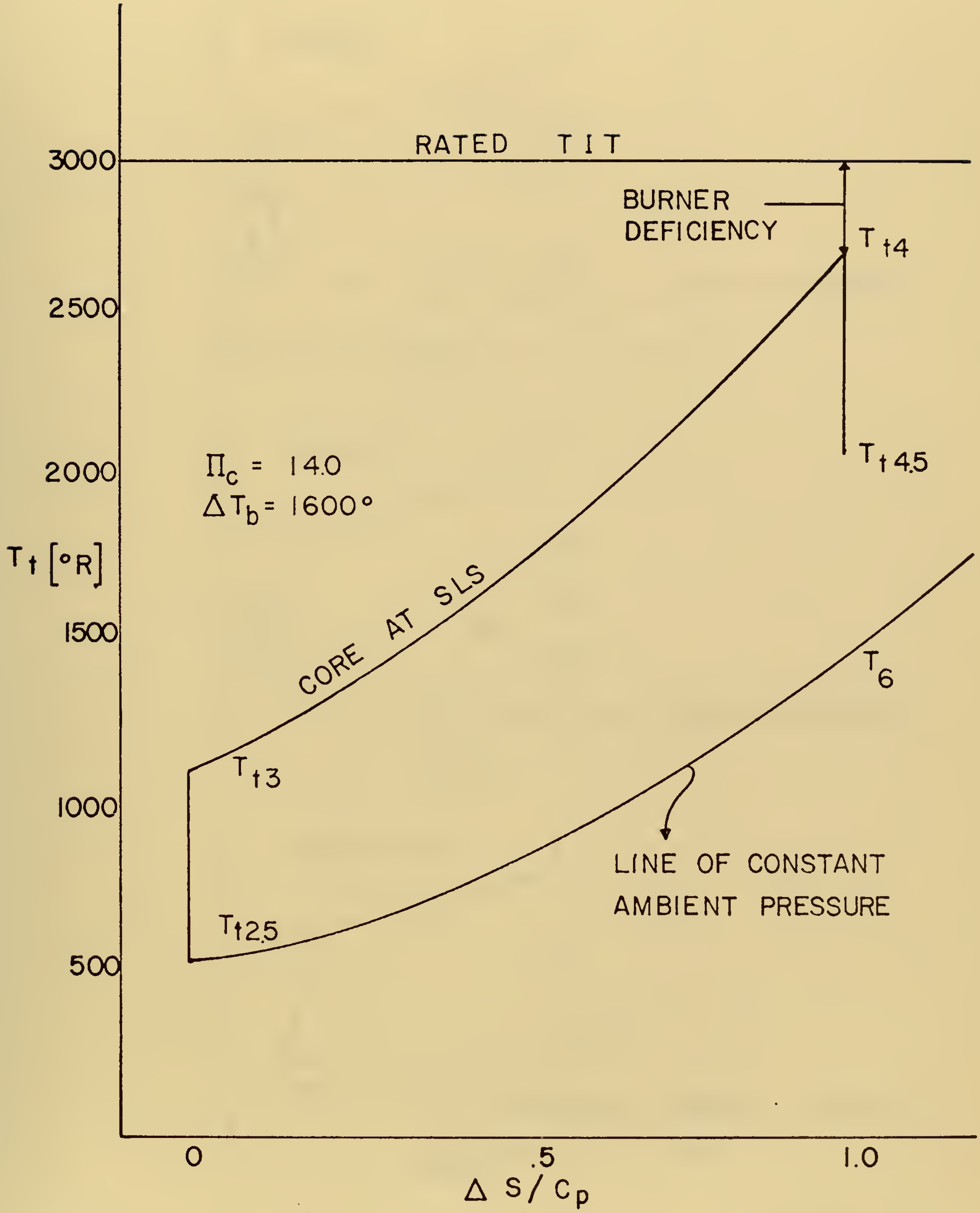
CORES IN SAME MISSION
WITH EQUAL Π_c BUT
DIFFERENT TIT.

FIG. 7



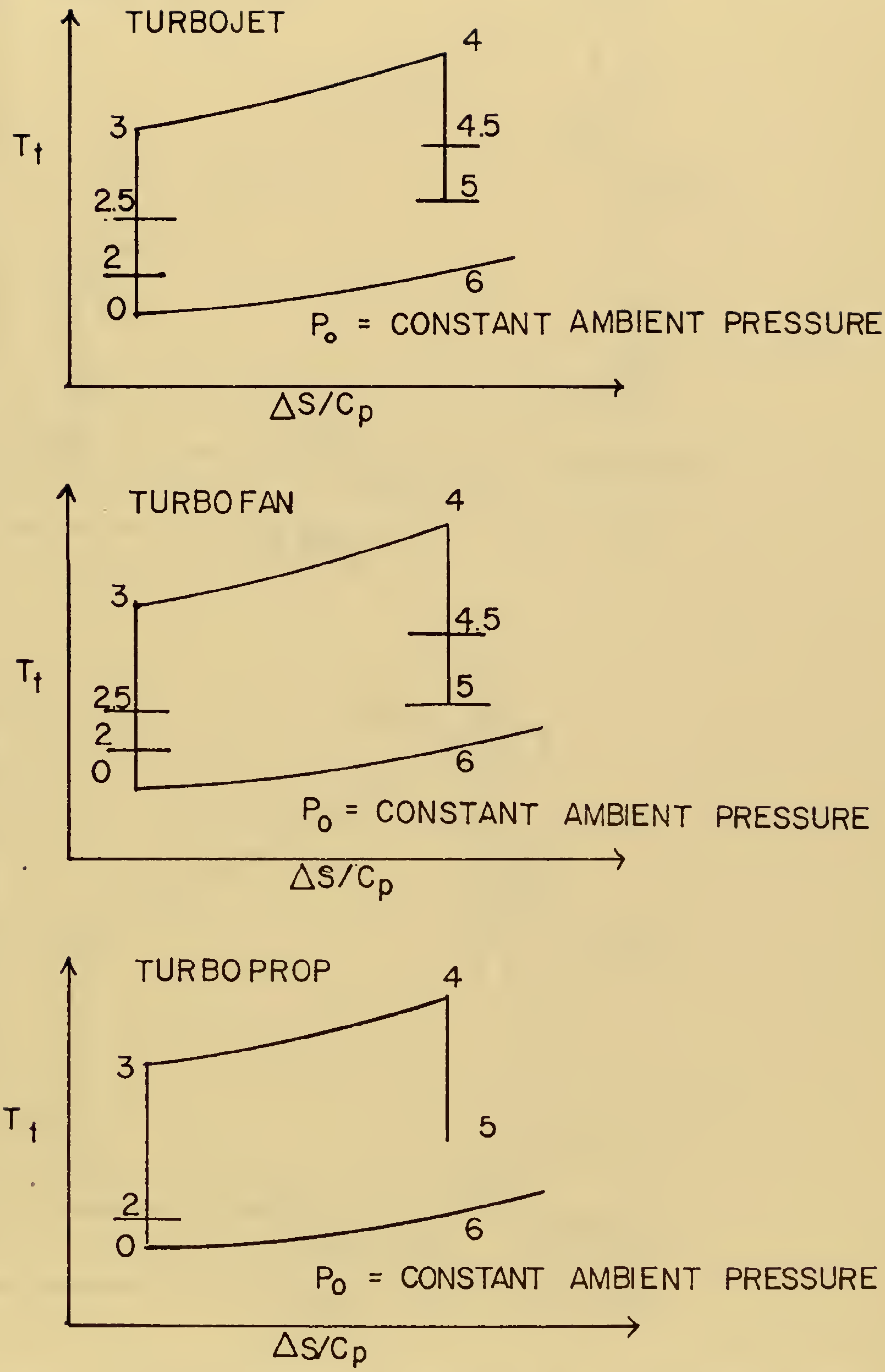
CORES WITH SAME TIT
BUT DIFFERENT Π_c

FIG. 8



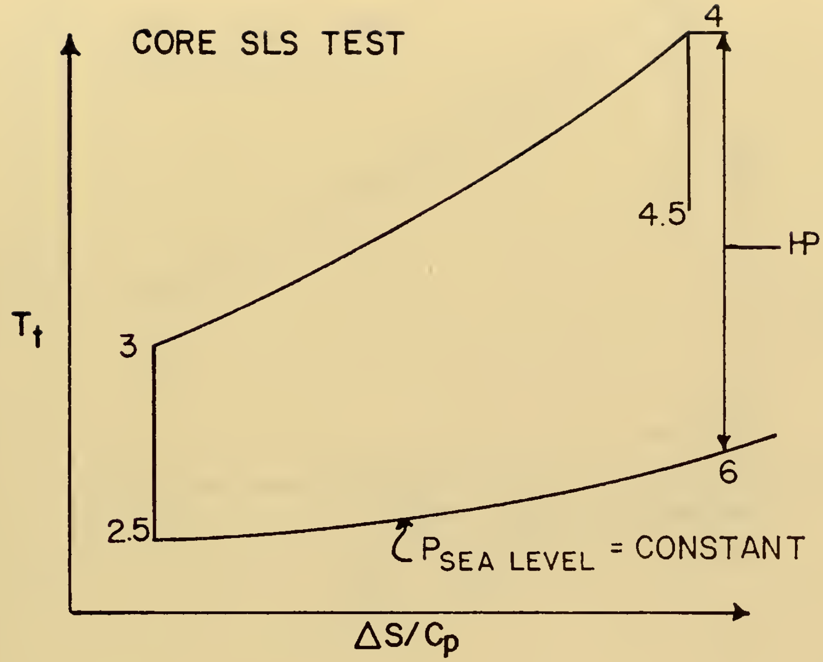
CORE BURNER DEFICIENCY

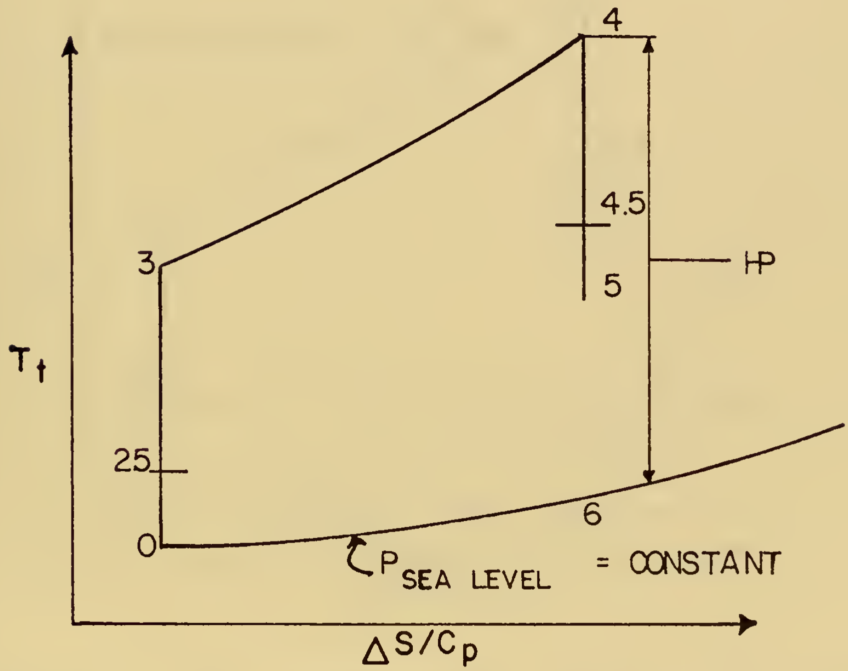
FIG. 9



THERMODYNAMIC CYCLES FOR GAS TURBINES

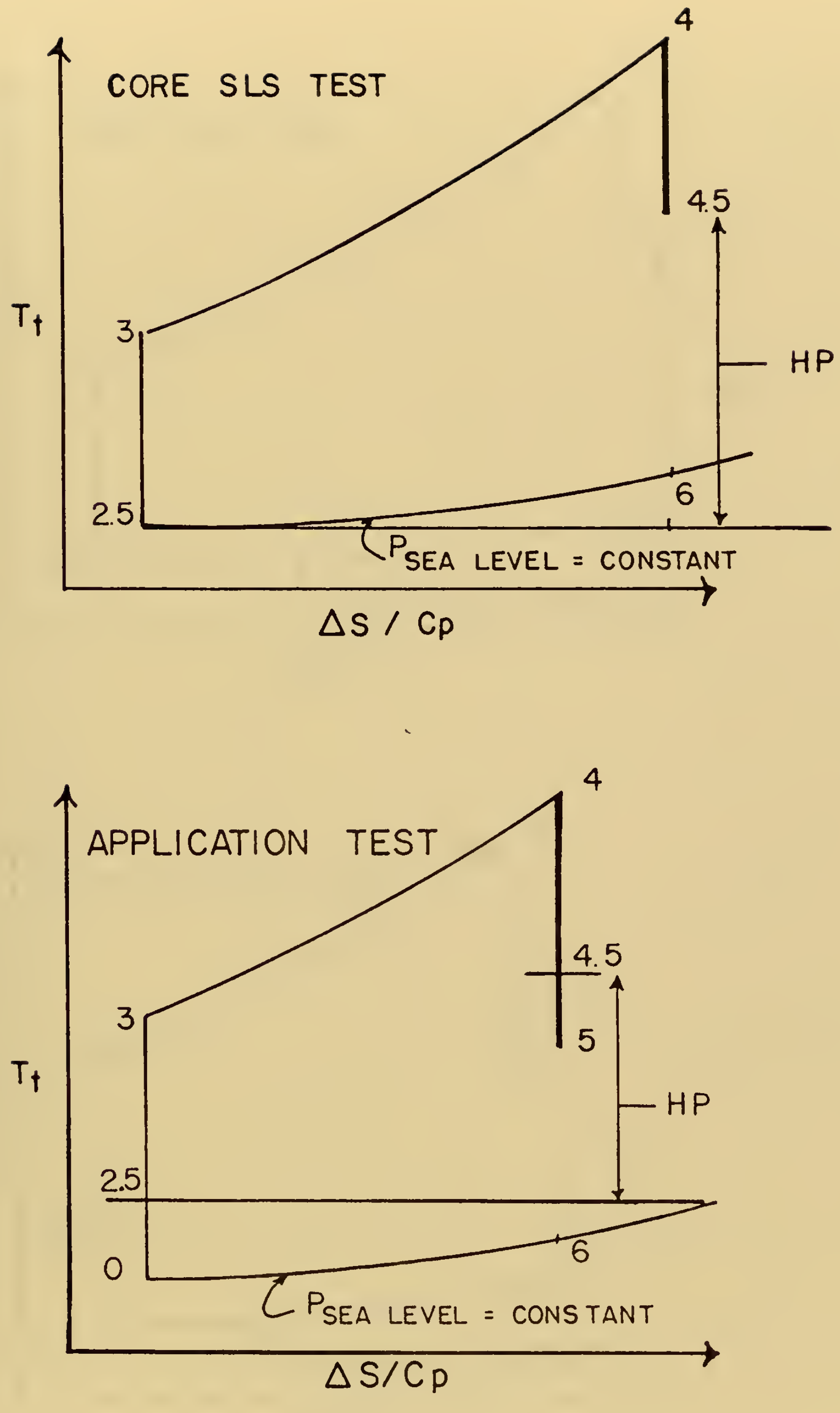
FIG. 10





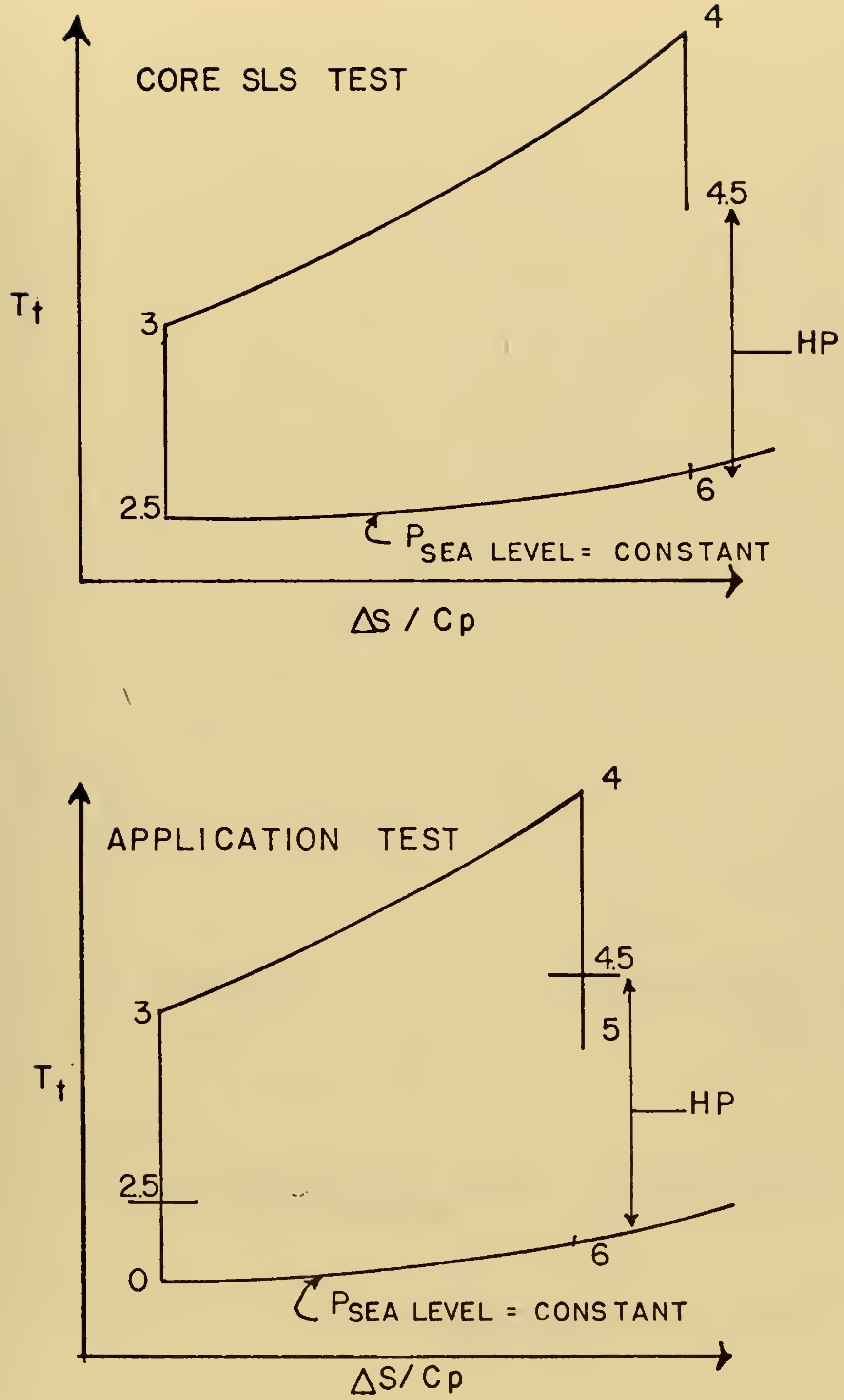
SPECIFIC GAS HORSEPOWER BASED ON TEMPERATURE DIFFERENCE BETWEEN BURNER OUTLET AND EXHAUST

FIG. 11



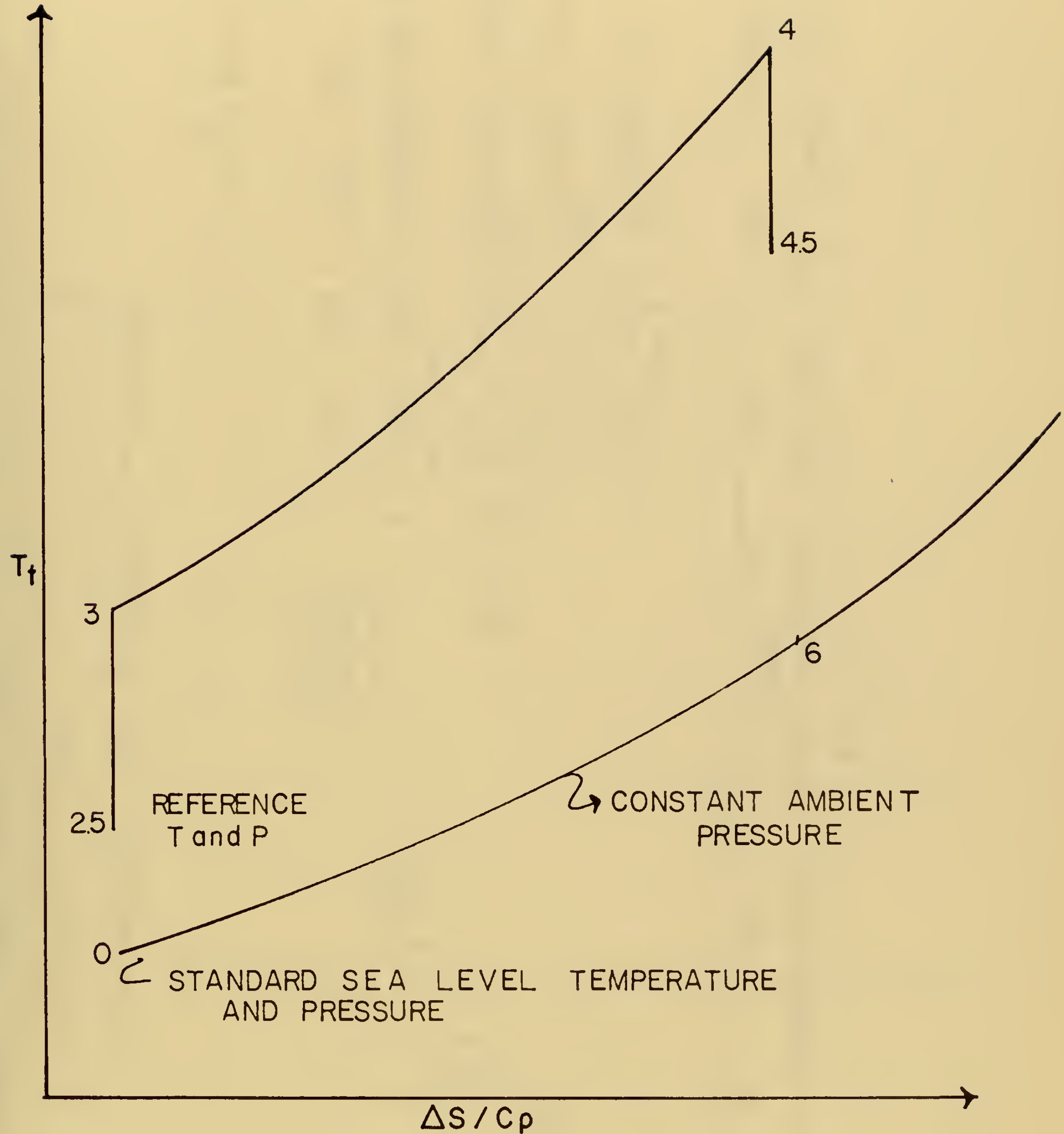
SPECIFIC GAS HORSEPOWER BASED ON TEMPERATURE AND PRESSURE DIFFERENCE BETWEEN CORE TURBINE OUTLET AND COMPRESSOR INLET.

FIG. 12



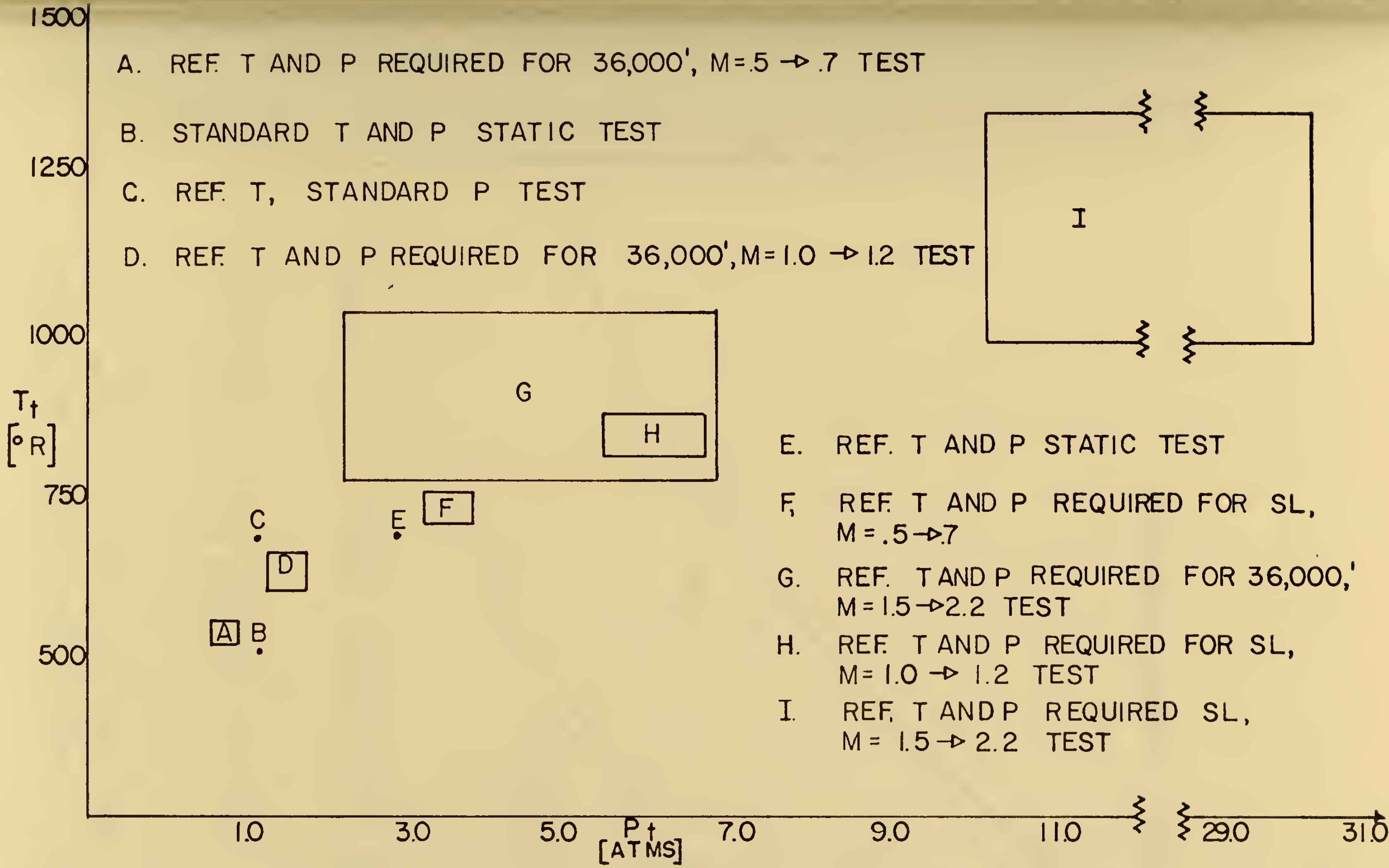
SPECIFIC GAS HORSEPOWER BASED ON TEMPERATURE DIFFERENCE BETWEEN CORE TURBINE OUTLET AND EXAUST EXPANDED TO AMBIENT PRESSURES

FIG. 13



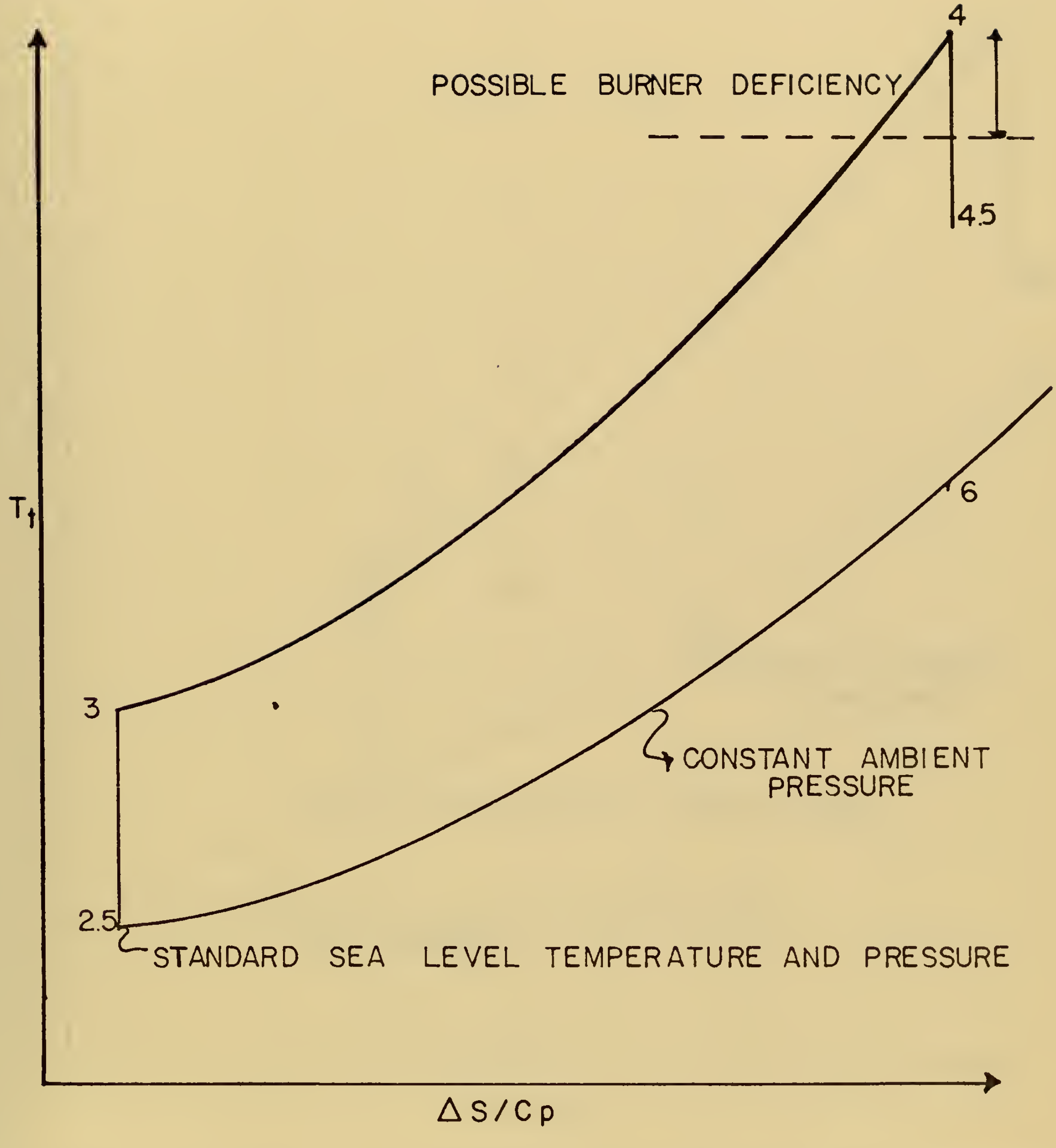
CORE TEST AT REFERENCE TEMPERATURE AND PRESSURE

FIG. 14



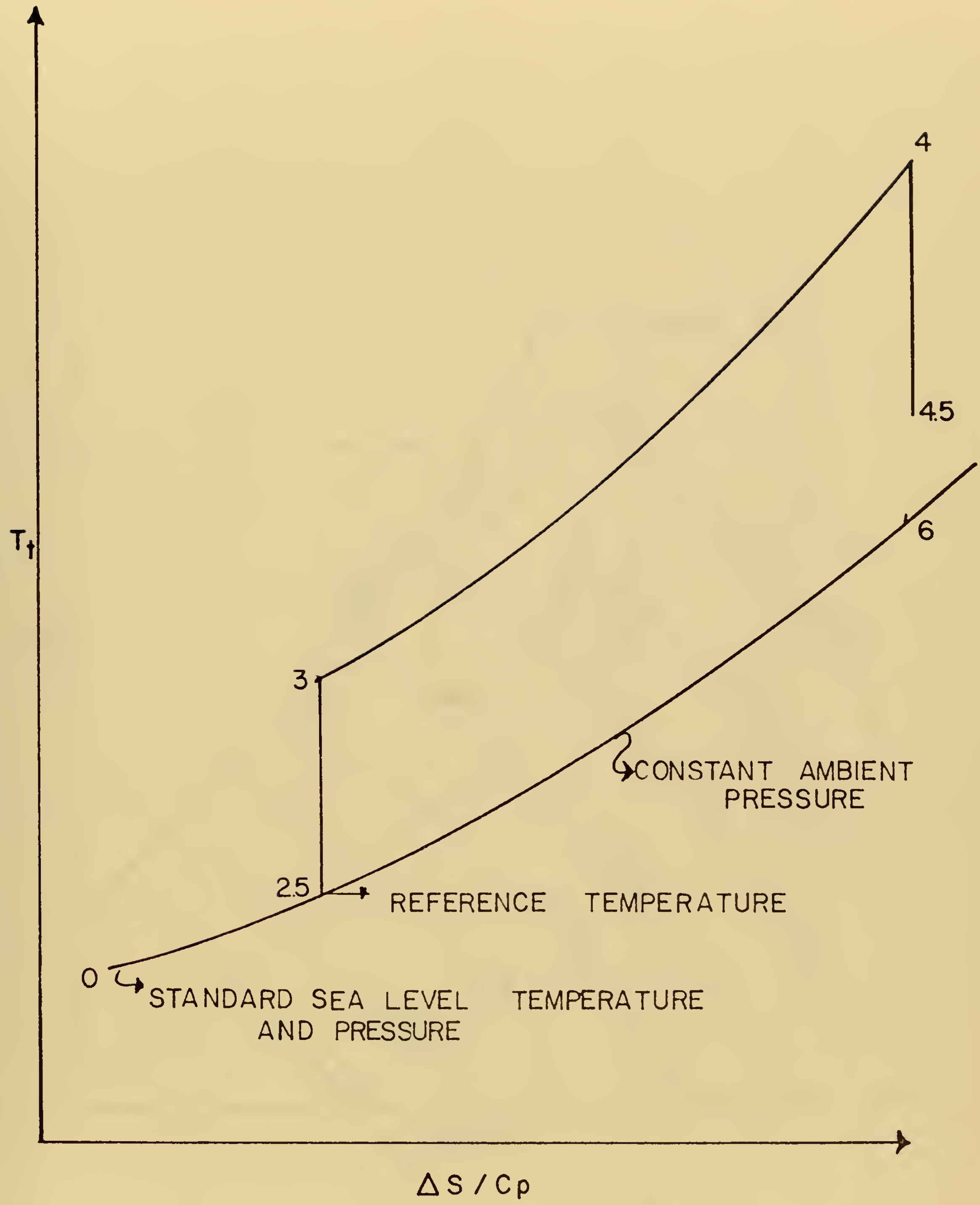
CORE INLET TEMPERATURE AND PRESSURE REQUIRED FOR VARIOUS TEST CONDITIONS

FIG. 15



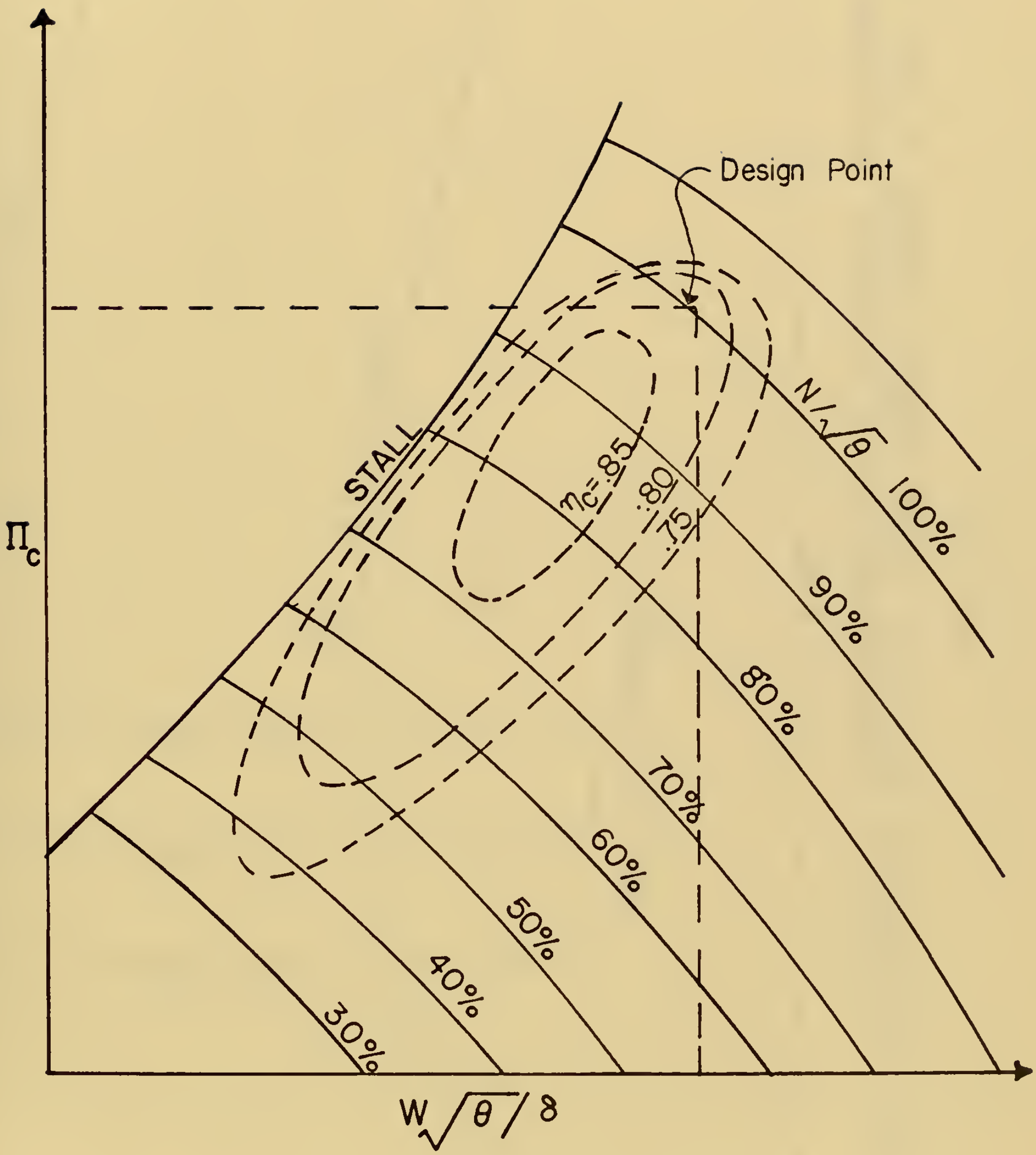
CORE TEST WITH STANDARD TEMPERATURE AND PRESSURE INLET CONDITIONS

FIG. 16



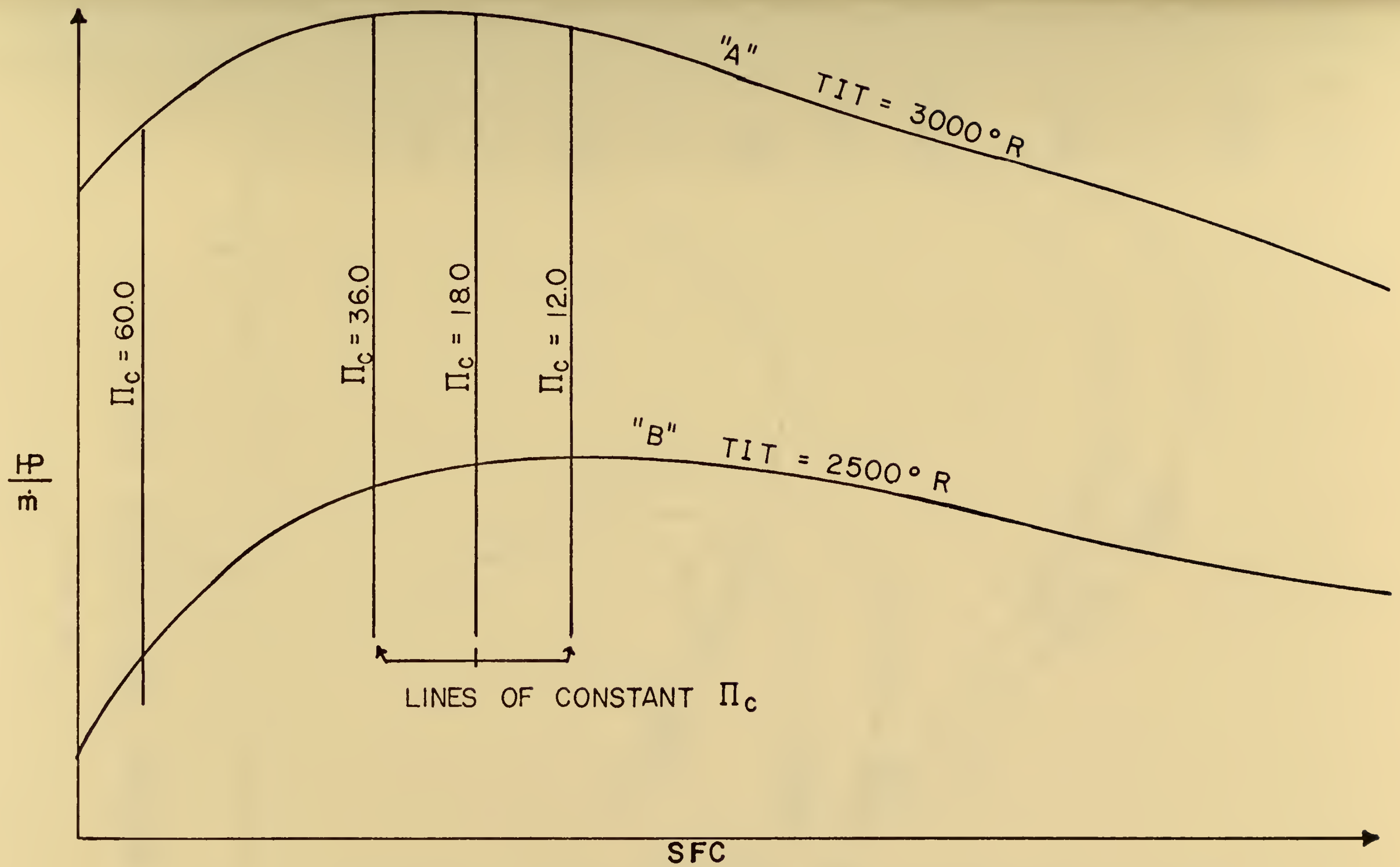
CORE TEST WITH REFERENCE TEMPERATURE AND STANDARD PRESSURE

FIG. 17



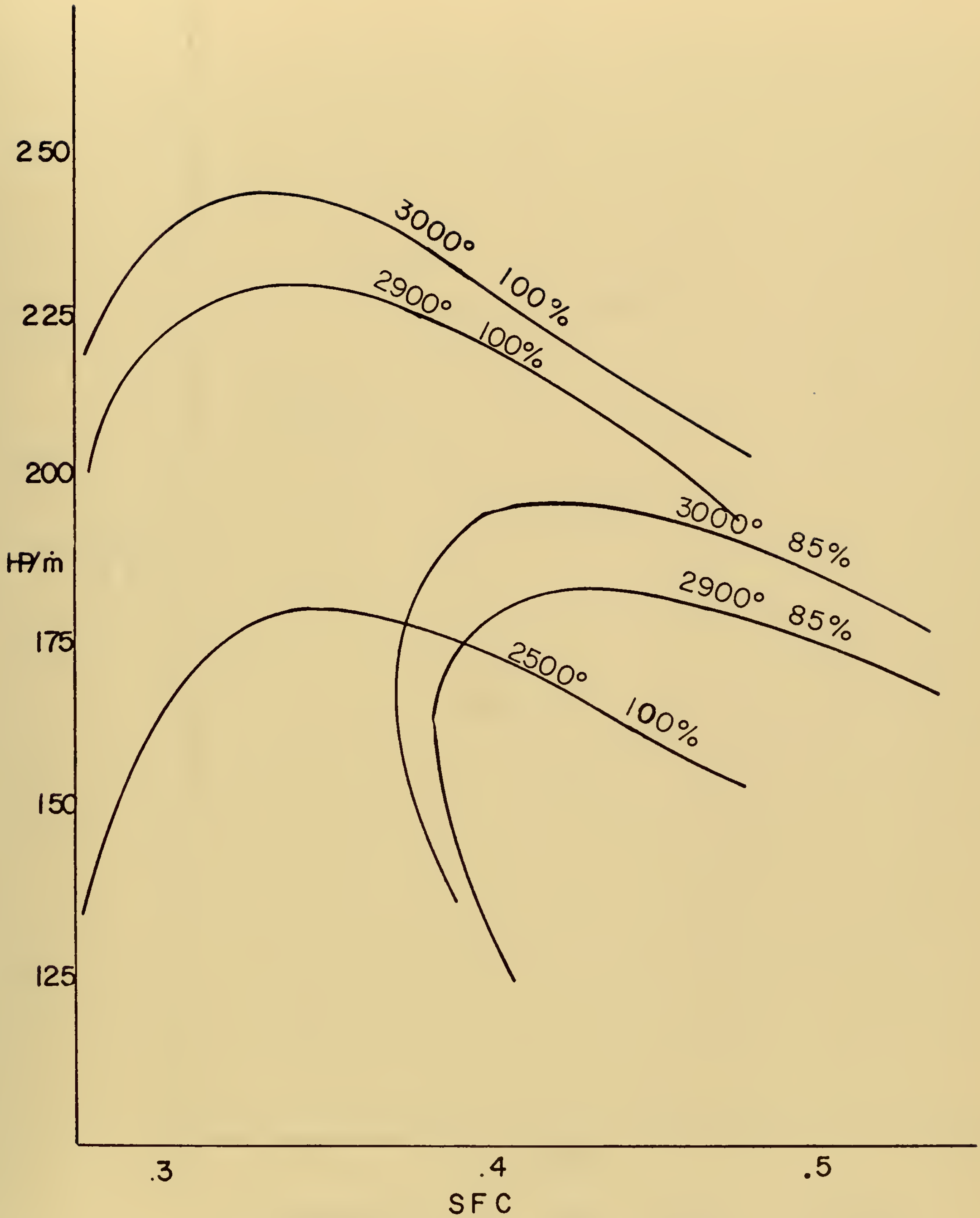
CORE COMPRESSOR MAP

FIG. 18



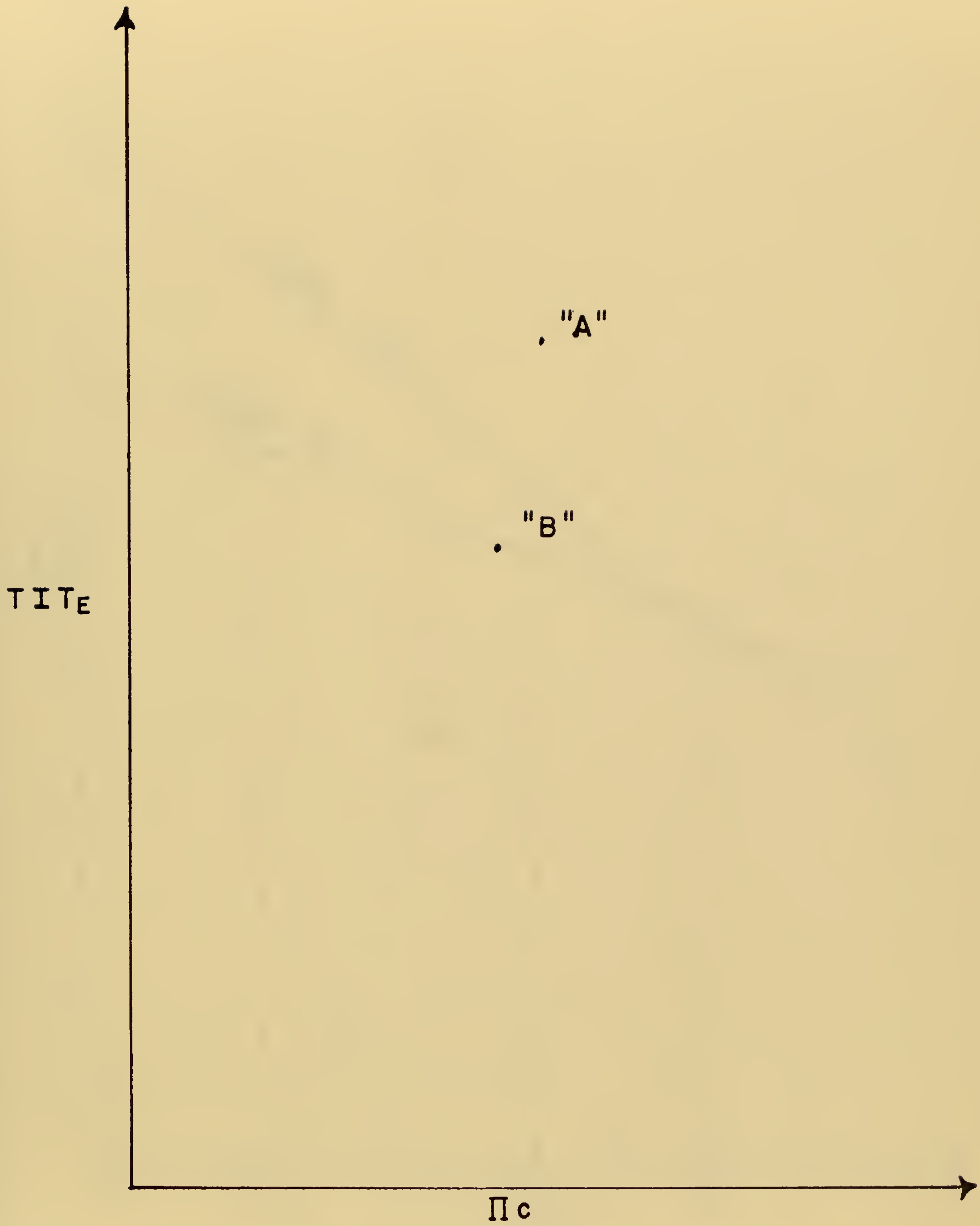
SPECIFIC HORSEPOWER AS A FUNCTION OF SPECIFIC
FUEL CONSUMPTION FOR TWO IDEAL CORES TESTED AT SEA LEVEL STATIC

FIG. 19



HP/ṁ VS. SFC FOR VARIOUS CORES

FIG. 20



RESULTS OF CORE TESTS PLOTTED IN THE $TIT_E - \Pi c$ PLANE

FIG. 21

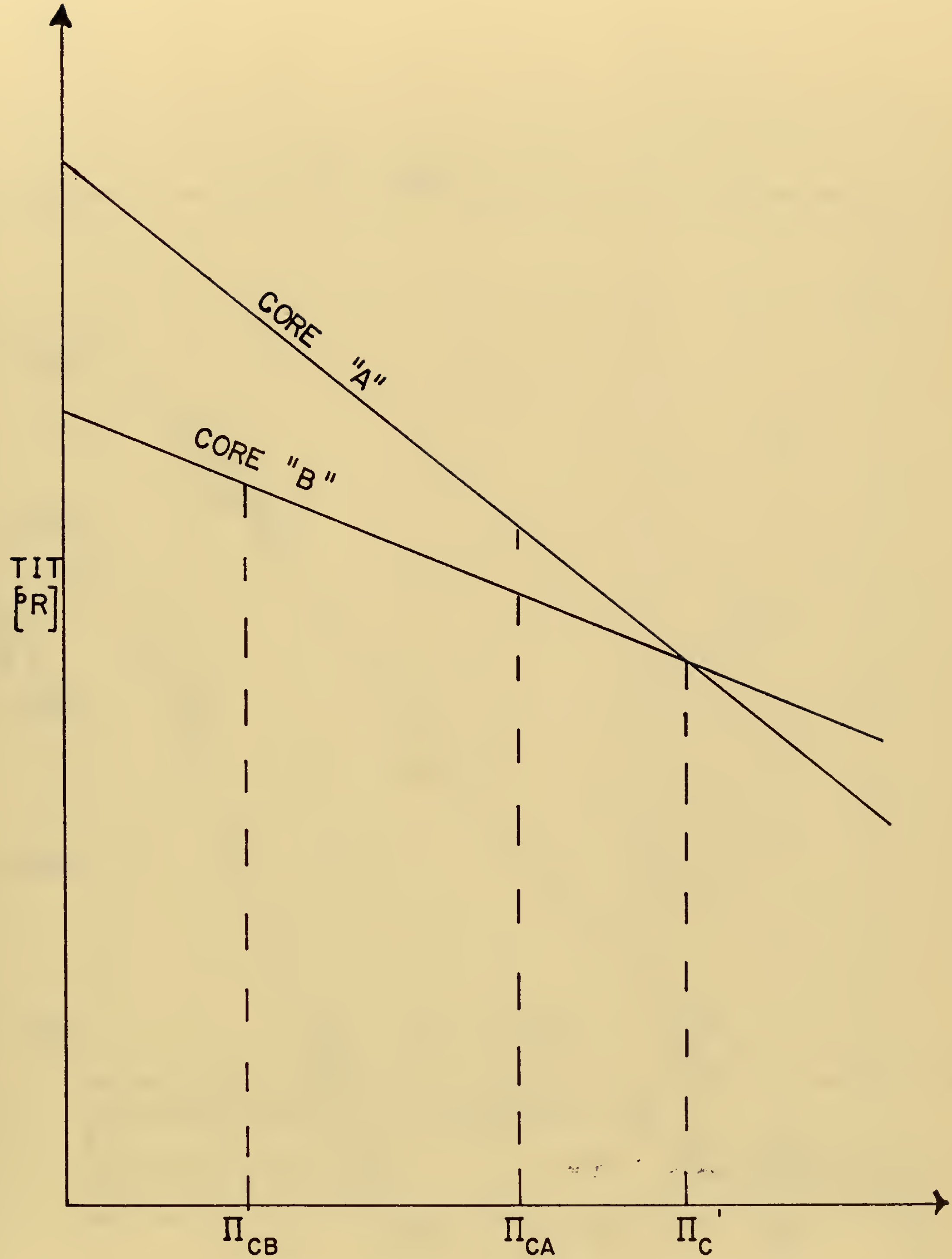
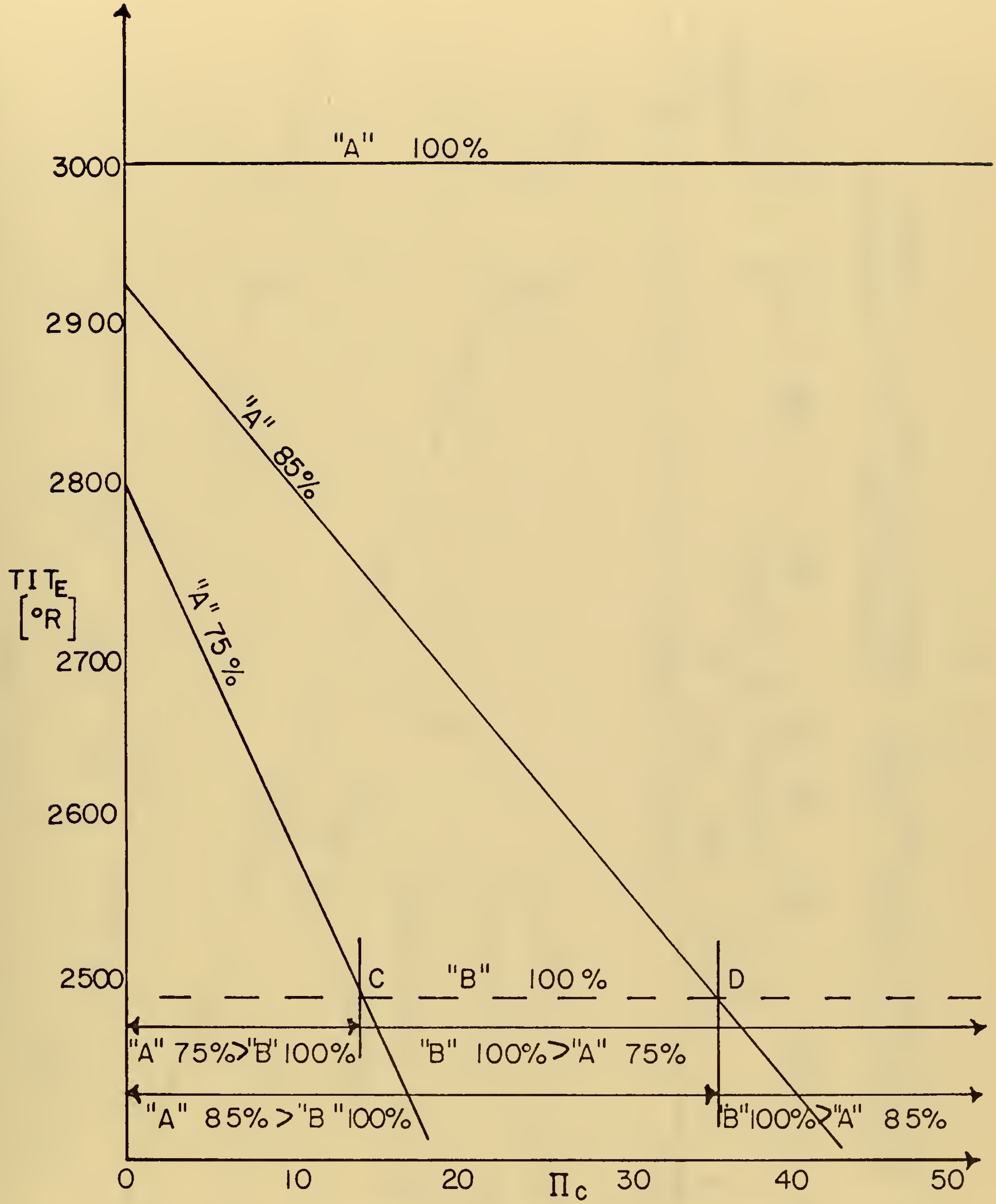


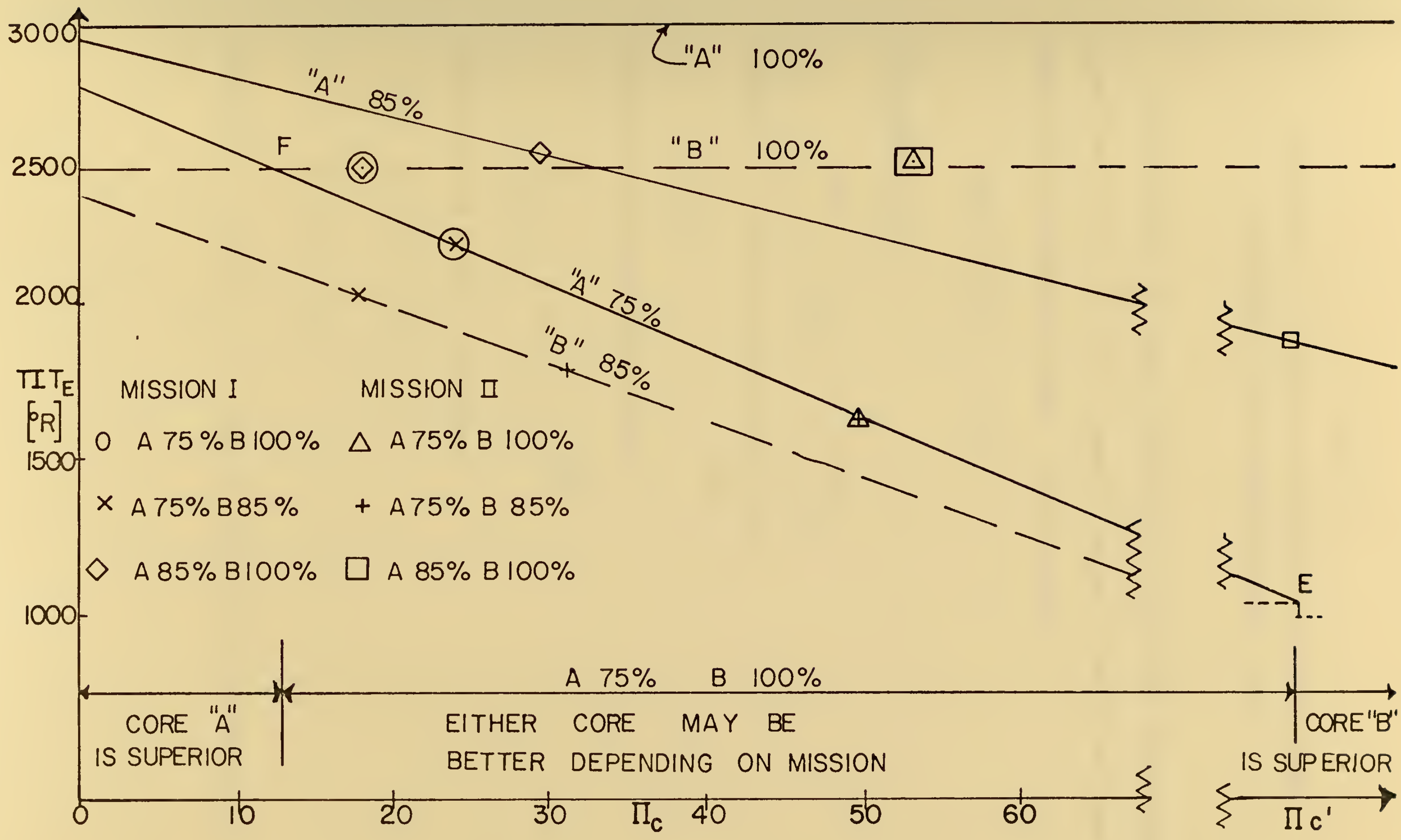
ILLUSTRATION OF CORE AND CRITICAL PRESSURE RATIOS

FIG. 22



EFFECTIVE TURBINE INLET TEMPERATURE AS A FUNCTION OF COMPRESSION RATIO FOR CORE "A" AT VARIOUS EFFICIENCIES AND AN IDEAL CORE "B"

FIG. 23



EFFECTIVE TURBINE INLET TEMPERATURE AS A FUNCTION OF COMPRESSION RATIO FOR VARIOUS CATAGORY 2 CORES

FIG. 24

APPENDIX A

DERIVATION OF EQUATIONS FOR SPECIFIC GAS HORSEPOWER AND SPECIFIC FUEL CONSUMPTION FOR A CORE

Starting with the basic definitions of HP/m and SFC as given by equations (9) and (12) this appendix derives formula for these parameters when losses are present. By definition,

$$HP/m = C_p \, (T_{t_{4.5}} - T_6) \tag{A1}$$

Rearranging (A1),

$$HP/m = C_p \, T_{t_{4.5}} \, (1 - T_6/T_{t_{4.5}}) \tag{A2}$$

Expansion to ambient pressure implies,

$$P_6 = P_o \tag{A3}$$

Using the isentropic relationship,

$$(T_6/T_{t_{4.5}}) = (P_o/P_{t_{4.5}})^{\frac{\gamma-1}{\gamma}} \tag{A4}$$

For sea level static conditions,

$$P_o = P_{t_o} = P_{t_{2.5}} \tag{A5}$$

$$T_o = T_{t_o} = T_{t_{2.5}} \tag{A6}$$

Writing $P_{t_{4.5}}$ and $T_{t_{4.5}}$ in terms of pressure and temperature ratios,

$$P_{t_{4.5}} = P_{t_o} \, \pi_c \, \pi_b \, \pi_t \tag{A7}$$

$$T_{t_{4.5}} = T_{t_o} \tau_c \tau_b \tau_t \qquad (A8)$$

Substituting expressions (A7), (A8) and equations (A4) into (A2) yields,

$$HP/m = C_p T_{t_o} (\tau_c\tau_b\tau_t)\left[1 - \left(\frac{1}{\pi_c\pi_b\pi_t}\right)^{\frac{\gamma-1}{\gamma}}\right] \qquad (A9)$$

Equating compressor and turbine work,

$$C_p (T_{t_3} - T_{t_{2.5}}) = C_p (T_{t_4} - T_{t_{4.5}}) \qquad (A10)$$

Formulation of temperature ratio and simplification of (A10) results in,

$$\tau_t = 1 - \tau_c - 1/\tau^* \qquad (A11)$$

Turbine and compressor efficiencies can be written,

$$\eta_t = (1 - \tau_t)/1 - \pi_t^{\frac{\gamma-1}{\gamma}} \qquad (A12)$$

$$\eta_c = (\pi_c^{\frac{\gamma-1}{\gamma}} - 1)/(\tau_c - 1) \qquad (A13)$$

Solving (A12) for π_t,

$$\pi_t = \left[\frac{\tau_t - 1}{\eta_t} + 1\right]^{\gamma/\gamma-1} \qquad (A14)$$

Substituting equation (A11) for τ_t in (A14),

$$\pi_t = \left[\frac{1 - \tau_c}{\tau^* \eta_t} + 1\right]^{\gamma/\gamma-1} \qquad (A15)$$

But from (A13),

$$1 - \tau_c = \frac{1 - \pi_c^{\frac{\gamma-1}{\gamma}}}{\eta_c} \qquad (A16)$$

Therefore,

$$\pi_t = \left[\frac{1 - \pi_c^{\frac{\gamma-1}{\gamma}}}{\tau^* \eta_c \eta_t} + 1 \right]^{\frac{\gamma}{\gamma-1}} \tag{A17}$$

Combining equations (A11) and (A16),

$$\tau_t = 1 - \frac{\pi^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c \tau^*} \tag{A18}$$

Substituting equations (A17) and (A18) into (A9) and recalling that $\tau^* = \tau_c \tau_b$, yields an expression for gas horsepower with losses in terms of compressor and turbine efficiencies,

$$\frac{HP}{m} = C_p T_{t_o} \quad \tau^* \left(1 - \left[\frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c \tau^*}\right]\right)\left(1 - \frac{1}{(\pi_c \pi_b)^{\gamma-1/\gamma}\left(1 - \frac{\pi_c^{\gamma-1/\gamma}}{\tau^* \eta_c \eta_t}\right)}\right) \tag{A19}$$

Similarly starting with the definition of SFC,

$$SFC = C_p \, (T_{t_4} - T_{t_3})/Q \; (HP/m) \tag{A20}$$

Forming temperature ratios,

$$SFC = C_p \, T_{t_4} \, (1 - T_{t_3}/T_{t_4})/Q(HP/m) \tag{A21}$$

Substituting τ^* into (A21),

$$SFC = C_p \, T_{to} \, \tau^* \, (1 - \tau_c/\tau^*)/Q(HP/m) \tag{A22}$$

Solving equation (A13) for τ_c and substituting into (A22) yields the following equation,

$$SFC = C_p \, T_{t_o} \left[\tau^* - \left(1 + \frac{\pi_c^{\frac{\gamma-1}{\gamma}} - 1}{\eta_c} \right) \right] / Q(HP/m) \qquad \text{(A23)}$$

APPENDIX B

CALCULATION PROCEDURE FOR EFFECTIVE TURBINE INLET TEMPERATURE

The following briefly describes the workings of computer program "TIT_E". For a specified set of inlet conditions, TIT, compressor, combustor and turbine efficiencies, the engine compression ratio is allowed to vary from 2.0 to 60.0 in 30 even increments. For each compression ratio values, of HP/m and SFC were computed using equations (26) and (27). The efficiencies in these equations were then set equal to unity and the resulting equations were solved for the ideal compression ratio and TIT_E.

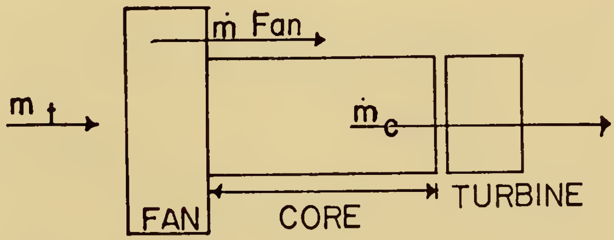
For a fixed value of TIT and efficiencies the program printed values of TIT, efficiency, actual compression ratio, ideal compression ratio, HP/m, SFC, and TIT_E as a function of actual compression ratio. In the ensuing steps the specified value of TIT was maintained, but turbine and compressor efficiencies were decreased in 0.05 increments over the range $.75 \leq \eta \leq 1.00$. Finally, TIT was allowed to vary from $3000^{\circ}R$ to $2500^{\circ}R$ in 100° steps. The program contains a glossary of terms and pertinent comment cards which further assist in understanding the calculations and associated output.

APPENDIX C

DERIVATION OF PROPULSION WEIGHT FORMULA FOR A TURBOFAN

The propulsion weight model for a turbofan is developed similarly to that of a turbojet. The major modification is that the engine weight is expressed as a function of core weight and by-pass ratio B.

The following figure shows the nomenclature used in the development.



The by-pass ratio is defined,

$$\beta = m_{fan}/m_c \tag{C1}$$

and total mass flow rate,

$$m_t = m_c + \dot{m}_{fan} \tag{C2}$$

Solving for m_c and m_{fan} in terms of β and m_t,

$$m_c = m_t/(1 + \beta) \tag{C3}$$

$$m_{fan} = m_t/(1 + \frac{1}{\beta}) \tag{C4}$$

Expressions for the weight of the core and the weight of the fan and its turbine,

$$W_C = C_2 m_2 = C_2 m_t/(1 + \beta) \tag{C5}$$

$$W_{F\ \&\ T} = C_2{}' \ m_{fan} = C_2{}' \ m_t/(1 + \frac{1}{\beta}) \tag{C6}$$

Adding (C5) and (C6) yields engine weight,

$$W_E = W_C + W_{F\,\&\,T} \tag{C7}$$

$$= \left[(C_2/(1 + \beta) + C_2'/(1 + \tfrac{1}{\beta}) \right] \dot{m}_t \tag{C8}$$

$$= \dot{m}_t \; (C_2 + \beta\, C_2')/(1 + \beta) \tag{C9}$$

Reference 8 yields values of the ratio of the core weight to total engine weight, where in this thesis R has replaced the symbol Kgg,

$$R = W_C/(W_C + W_{F\,\&\,T}) = C_2/(C_2 + \beta C_2') \tag{C10}$$

Solving for C_2'/C_2 ,

$$C_2'/C_2 = (1 - R)/\beta \tag{C11}$$

Manipulating equation (C9) into this form,

$$W_E = C_2 \left[\frac{(1 + \beta)\; C_2'/C_2}{1 + \beta} \right] \tag{C12}$$

Substituting (C11) into (C12),

$$W_E = \dot{m}_t \; C_2(2\text{-}R)/(1+\beta) \tag{C13}$$

Equation (C13) allows engine weight to be expressed as a function of the core weight and the by-pass ratio, two well known parameters. Utilizing equation (C13) Propulsion weight for a turbofan can be written,

$$\text{PROP WT.} = \left(\frac{\text{A/C WT.}}{\text{L/D}}\right) \left\{ \left[\frac{\dot{m}_t}{F} \left(C_1 + C_3 + \frac{C_2(2\text{-}R)}{1+\beta}\right) \right] + (\text{SFC})(\text{RANGE}/a_o\, M_o) \right\} \tag{C14}$$

It should be noted that values of C_i will be different than those determined for a turbojet.

APPENDIX D

CALCULATION PROCEDURE FOR PROPULSION WEIGHT

The computer program labeled "PROPULSION WEIGHT" computes propulsion weight as defined in equation (29). The program initially fixes mission parameters (Mach number, ambient temperature at altitude, and duration), TIT and efficiencies. Engine compression ratio is allowed to vary with each ratio producing a unique value of propulsion weight. The program has built-in capability, via DO loops, to vary TIT, efficiencies and duration. The program prints duration, efficiencies, compression ratio, F/m, SFC and propulsion weight. A glossary of terms and relevant comment cards are provided to assist in interpreting the program.

```
C
C
C
C         COMPUTER  PROGRAM  I  'EFFECTIVE  TIT'
C
C
C
C
C
C         GLOSSARY  OF  TERMS:
C
C               TO=AMBIENT  TEMPERATURE
C               PIB=COMBUSTOR  PRESSURE  RATIO  (PT4/PT3)
C               GAMMA=RATIO  OF  SPECIFIC  HEATS  (CP/CV)
C               CP=SPECIFIC  HEAT  AT  CONSTANT  PRESSURE
C               H=HEATING  VALUE  (BTU/LBM)
C               T4=TURBINE  INLET  TEMPERATURE
C               EFFCPR=COMPRESSOR  EFFICIENCY
C               EFFCPR=COMPRESSOR  EFFICIENCY
C               PIC=COMPRESSOR  RATIO
C               TC=COMPRESSOR  TEMPERATURE  RATIC
C               PIT=TURBINE  PRESSURE  RATIO  PT4.5/PT4
C               TT=TURBINE  TEMPERATURE  RATIO
C               T4EFFT=EFFECTIVE  TURBINE  INLET  TEMPERATURE
C               HPACTL=HORSEPOWER  WITH  LOSSES
C               SFC=SPECIFIC  FUEL  CONSUMPTION  WITH  LOSSES
C               TCIDEL=IDEAL  COMPRESSOR  TEMPERATUTRE  RATIO
C               PICIDL=IDEAL  COMPRESSOR  PRESSURE  RATIO
C               TSTARI=NO  LOSS  VALUE  OF  TT4/TO
C               T4EFFT=EFFECTIVE  TURBINE  INLET  TEMPERATURE
C
C
C             THIS  PROGRAM  EQUATES  HP  AND  SFC  FOR  A  GIVEN
C         ACTUAL  CORE  TO  AN  IDEAL  CORE(NO  LOSSES).THE  TIT
C         (EFFECTIVE  TIT)  TO  SATISFY  THE  RESULTING  EQUAL
C         VALUES  OF  HP  AND  SFC  IS  CALCULATED.
C
C
C
C
C
      WRITE(6,100)
      TO=519.0
      H=18900.0
      CP=.24
      GAMMA=1.4
      Z=(GAMMA-1.0)/GAMMA
      Y=1.0/Z
      PIB=1.0
```

```
C
C
C
C
C     .....ACTUAL RATED TIT....
      T4=2600.0
C     .....TIT LOOP....
      DO 13 L=1,2
      T4=T4-100.0
      TSTAR=T4/TO
      EFFTBR=1.05
      EFFCPR=1.05
C     .....DO LOOP TO VARY EFFICIENCIES....
      DO 10 I=1,6
      EFFCPR=EFFCPR-.05
      EFFTBR=EFFTBR-.05
      PIC=4.0
C     .....COMPRESSION RATIO DO LOOP....
      DO 11 J=1,34
      PIC=PIC+2.0
      TC=(PIC**Z-1.0)/EFFCPR+1.0
      PIT=((1.0-TC)/(TSTAR*EFFTBR)+1.0)**Y
      TT=1.0-(TC-1.0)/TSTAR
C     .....COMPUTATION OF ACTUAL HORSEPOWER....
      HPACTL=CP*T4*TT*(1.0-1.0/(PIC*PIB*PIT)**Z)
C     .....COMPUTATION OF ACTUAL SFC....
      SFCACT=(3600.0*CP*T4*(1.0-TC/TSTAR))/(H*HPACTL)
C     .....CORRESPONDING IDEAL COMPRESSOR TEMPERATURE RATIO..
      TCIDEL=SFCACT/(SFCACT-(3600.0/H))
      PICIDL=TCIDEL**Y
      TSTARI=(TCIDEL/(TCIDEL-1.0))*(HPACTL/(CP*TO))+TCIDEL
      WRITE(6,101)T4,EFFCPR,PIC,PICIDL,T4EFFT,HPACTL,SFCACT
   11 CONTINUE
   10 CONTINUE
   13 CONTINUE
  100 FORMAT('1',11X,'TIT',14X,'EFFICIENCY',10X,'PIC',14X
     X'PICIDL',10X,'TITE',15X,'HP',15X,'SFC',//)
  101 FORMAT('0',10X,F6.0,14X,F5.2,12X,F5.1,12X,F6.2,10X
     XF6.0,11X,F6.1,13X,F6.3)
      STOP
      END
```

```
C
C
C
C
C     COMPUTER PROGRAM II 'PROPULSION WEIGHT'
C
C
C
C
C     GLOSSARY OF TERMS:
C
C           B=BY PASS RATIO
C           AMACH=MISSION MACH NUMBER
C           T4=TURBINE INLET TEMPERATURE
C           TO=AMBIENT TEMPERATURE
C           H=HEATING VALUE (BTU/LB)
C           CP=SPECIFIC HEAT AT CONSTANT PRESSURE
C           C1=INLET WEIGHT CONSTANT
C           C2=ENGINE WEIGHT CONSTANT
C           C3=NOZZLE WEIGHT CONSTANT
C           DURATN=MISSION DURATION IN HOURS
C           DL=LIFT TO DRAG RATIO
C           GAMMA=RATIO OF SPECIFIC HEATS (CP/CV)
C           R=UNIVERSAL GAS CONSTANT
C           AO=SPEED OF SOUND AT AMBIENT CONDITIONS
C           TRAM=RAM TEMPERATURE RATIO TT2/TO
C           U90= RATIO OF VELOCITY AT 9/VELOCITY AT 0
C           EFFCPR=COMPRESSOR EFFICIENCY
C           EFFTBR=TURBINE EFFICIENCY
C           PIC=COMPRESSOR RATIO
C           TC =COMPRESSOR TEMPERATURE RATIC
C           TT=TURBINE TEMPERATURE RATIO
C           U80=RATIO OF VELOCITY AT 8/VELOCITY AT 0
C           SIMPLE=SPECIFIC IMPULSE (F/MG)
C           SFC=SPECIFIC FUEL CONSUMPTION
C
C
C
C
C
C
C
C        THIS PROGRAM COMPUTES VALUES OF PROPULSION
C     WEIGHT BASED ON A MISSION PROFILE,TIT,ENGINE
C     COMPRESSION RATIO,TURBINE AND COMPRESSOR
C     EFFICIENCIES.
```

```
C
C
C
C
      WRITE(6,100)
C     ....MISSION PARAMETERS....
      B=0.0
      AMACH=1.5
      T4=3000.0
      TO=389.99
      H=18900.0
      CP=.24
      C1=3.0
      C2=20.0
      C3=3.0
      ACWGHT=50000.0
      DL=8.0
      RATIO=ACWGHT/DL
      G=32.17
      GPRIME=1.0/G
      GAMMA=1.4
      R=1717.0
      Z=(GAMMA-1.0)/GAMMA
      Y=1.0/Z
      TCFAN=1.0
      AO=SQRT(GAMMA*R*TO)
      TRAM=1.0+(GAMMA-1.0)/2.0*AMACH**2
      DURATN=.5
C     ....START DURATION DO LOOP....
      DO 12 K=1,3
      DURATN=5.0*DURATN
      U90=SQRT((TRAM*TCFAN-1.0)/(TRAM-1.0))
      TSTAR=T4/TO
      EFFCPR=1.05
      EFFTBR=1.05
C     ....START OF DO LOOP FOR EFFICIENCIES....
      DO 10 I=1,6
      EFFCPR=EFFCPR-.05
      EFFTBR=EFFTBR-.05
      PIC=0.0
C     ....START OF DO LOOP FOR COMPRESSION RATIO....
      DO 11 J=1,31
      PIC=PIC+2.0
      TC=PIC**Z-1.0/EFFCPR+1.0
      TT=1.0-(TRAM*((TC-1.0)+B*(TCFAN-1.0)))/(TSTAR)
      U80=SQRT(TSTAR*TT/(TRAM*(TRAM-1.0))*(TRAM-1.0/((1.0+EF
     XFCPR*(TC-1.0))*(1.0-(1.0-TT)/EFFTBR))))
C     ....COMPUTATION OF SPECIFIC IMPULSE ....
      SIMPLE=AO*AMACH*GPRIME*(U80/(1.0+B)+B*U90/(1.0+B)-1.0)
      RECPRI=1.0/SIMPLE
C     ....COMPUTATION OF SFC....
      SFC=(3600.0*CP*TC*(TSTAR-TRAM*TC))/(H*(1.0+B)*SIMPLE)
C     ....COMPUTAION OF PROPULSION WEIGHT....
      PROPWT=RATIO*(RECPRI*(C1+C3+C2)+(SFC*DURATN))
      WRITE(6,101)DURATN,EFFCPR,PIC,SIMPLE,SFC,PROPWT
   11 CONTINUE
   10 CONTINUE
   12 CONTINUE
  100 FORMAT('1',10X,'DURATION (HRS)',7X,'EFFICIENCY',7X,
     X'PIC',8X,'I',8X,'SFC',6X,'PROP WT. (LBS)',//)
  101 FORMAT('0',14X,F5.1,14X,F5.2,9X,F5.1,5X,F5.1,5X,F5.3,
     X7X,F7.0)
      STOP
      END
```

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13. ABSTRACT

Methods for comparing gas turbine engines where the thermodynamic cycle begins and ends in the atmosphere are well defined and documented. No such comparison technique(s) exists for the gas generator or core portion of the engine. The term gas generator or core refers to the high pressure compressor and turbine, and the combustor.

This thesis formulates gas generator performance parameters, develops methods of testing and data reduction necessary to obtain these parameters, establishes criteria for comparing two gas generators, and develops an analytical model to test the validity of the comparison method.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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Core						
ATEGG						
Performance Parameters						
Engine Comparison						

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B88417 Burns
c.1 Performance parameters for gas generator comparison.